Duality and Geometry Straightness, Characterization and Envelope

Jean-Marc Chassery

LIS Grenoble

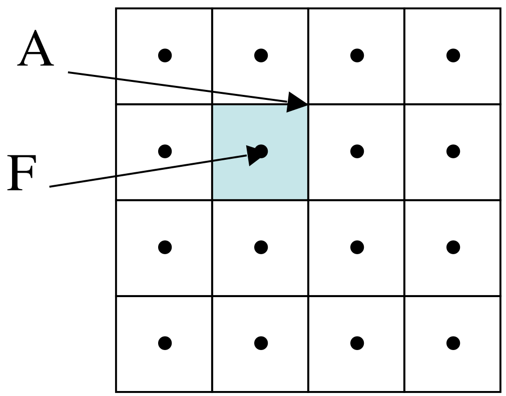
David Coeurjolly and Isabelle Sivignon

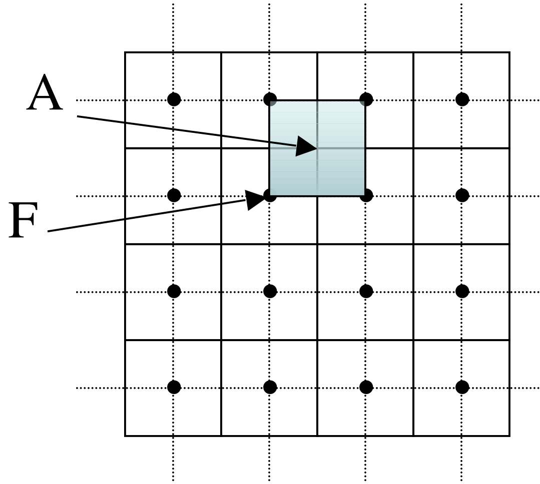
LIRIS Lyon

Outline

1. Introduction
2. Hough transform and discrete geometry
3. Preimage of a pixel, a set of pixels, a connected straight line
4. Preimage and Farey sequences
5. Extension in 3D
6. Duality versus polarity
7. Polygon enclosure and inclusion problems
8. Application to the convex approximation by interior.
9. Conclusion

1. Duality - Partitioning and meshes





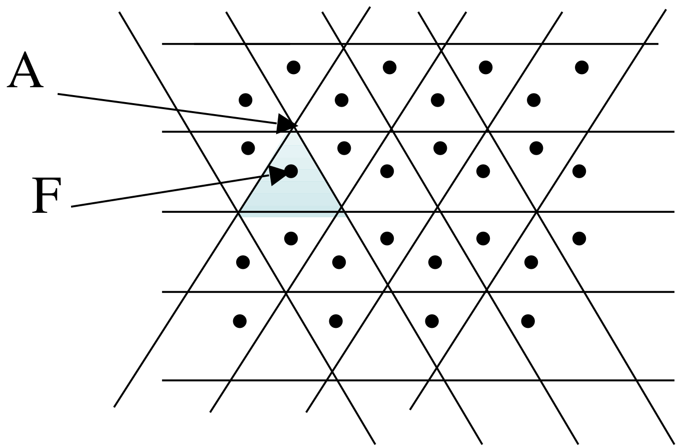
Let P be a partitionning.

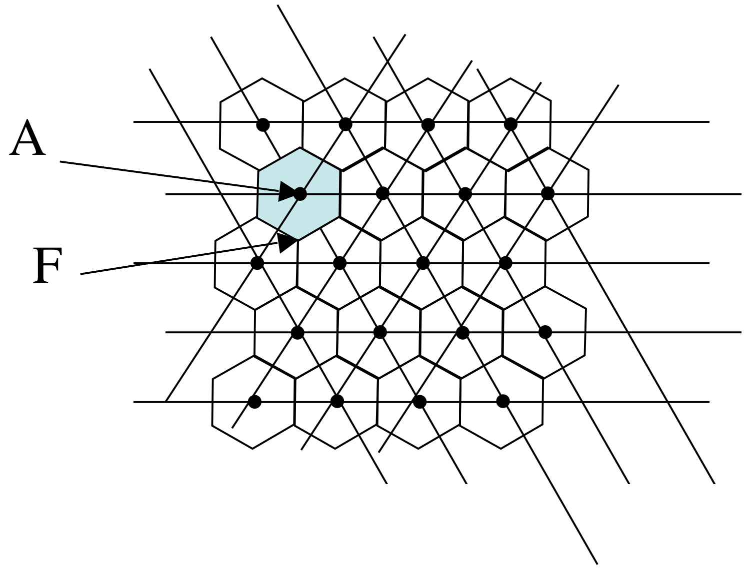
If A is a top point of P and F a face of P with $A \subset F$ then

in the dual of P, A is a face and F a top point with $F \subset A$.

The dual of a square partitionning is a square partitionning.

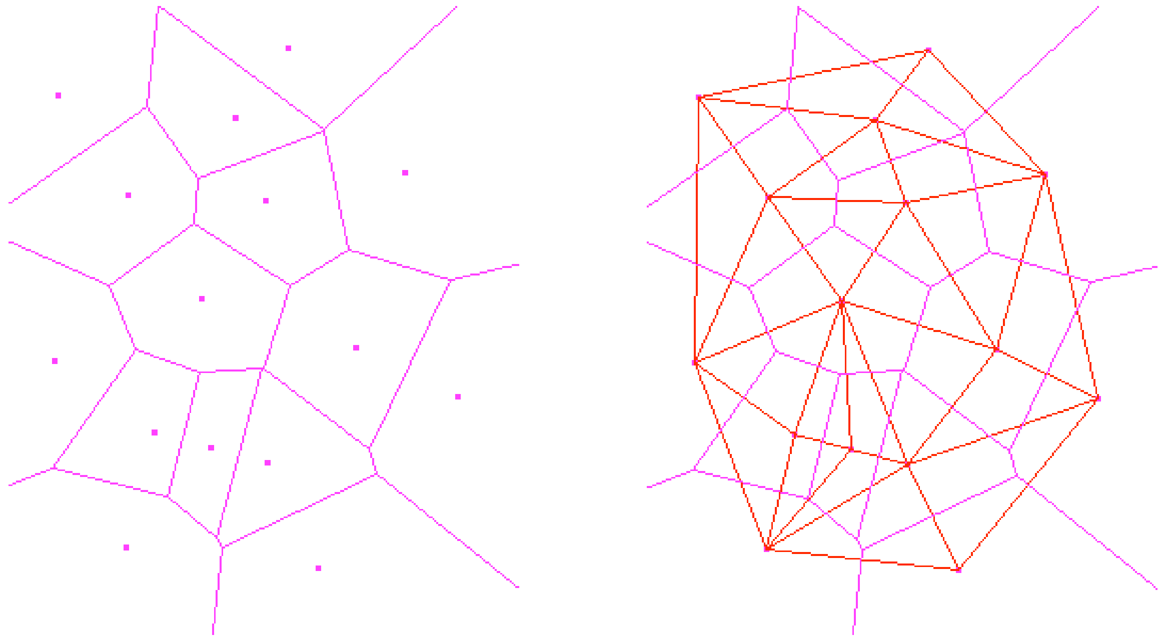
Duality - Partitioning and meshes





The dual of a triangular partitionning is an hexagonal partitionning.

Duality - Voronoi - Delaunay

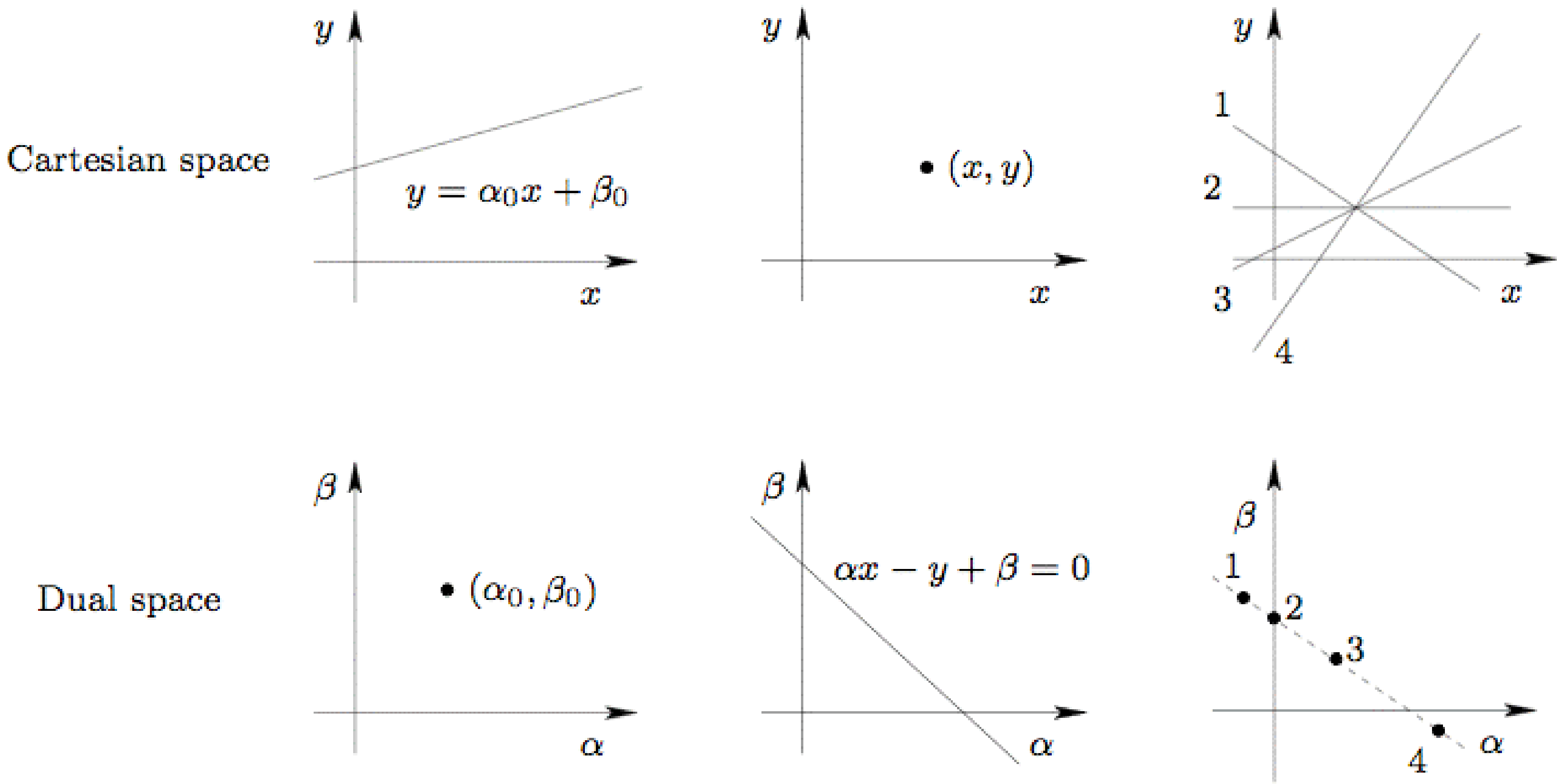


The dual of a Voronoï partitionning is a Delaunay triangulation.

Outline

1. Introduction
2. Hough transform and discrete geometry
3. Preimage of a pixel, a set of pixels, a connected straight line
4. Preimage and Farey sequences
5. Extension in 3D
6. Duality versus polarity
7. Polygon enclosure and inclusion problems
8. Application to the convex approximation by interior.
9. Conclusion

2. Hough Transform



- A point in the (x,y) space matches up with one curve in the dual space
- A point in the dual space matches up with one curve in the (x,y) space.
- Points lying on a same line in (x,y) space match up with concurrent curves in the dual space.
- Points on a same curve in dual space match up with concurrent straight lines in (x,y) space.

Hough in discrete geometry

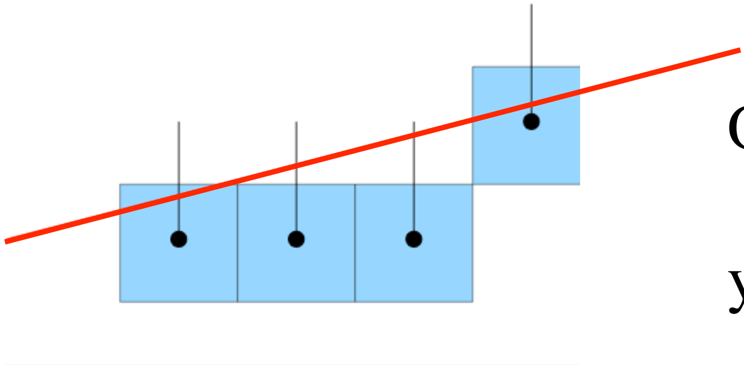
• Each point of parametric space can be seen either as a digital straight line or as an euclidean line, the discretization of which corresponding to the digital straight line.

• In discrete geometry a naive straight line is not given by an equality but by 2 inequalities:

$$0 \leq \alpha x - y + \beta < 1$$

So a pixel corresponds to the intersection of 2 half spaces in (α,β) parameter space.

Digitization process for a 8-connected Digital straight line segment



Object Boundary Quantization (OBQ) :

$y = \mathrm{Floor}\ (\alpha\ x + \beta)$ with $x \in Z$

From Pixel to strip in the dual space

Let $P(x_0, y_0)$ be a pixel and $L : y = \alpha x + \beta$ a straight line

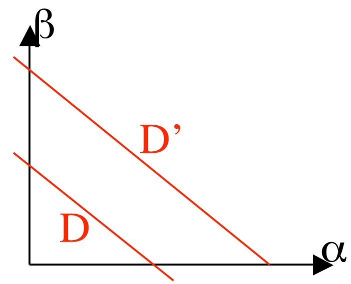
$$P \in OBQ(L) \Leftrightarrow y_0 = \text{Floor}(\alpha x_0 + \beta)$$

$$0 \leq \alpha x_0 + \beta - y_0 < 1$$

Strip noted by $B(x_0, y_0) = \{$ straight lines L ; $P \in OBQ(L)$ $\}$

(α, β) such that:

$$B(x_0, y_0) = \{\ 0 \leq \alpha x_0 + \beta - y_0 \ (D)$$
$$\text{and} \quad \alpha x_0 + \beta - y_0 < 1 \ (D')\ \}$$



Outline

1. Introduction
2. Hough transform and discrete geometry
3. Preimage of a pixel, a set of pixels, a connected straight line
4. Preimage and Farey sequences
5. Extension in 3D
6. Duality versus polarity
7. Polygon enclosure and inclusion problems
8. Application to the convex approximation by interior.
9. Conclusion

Preimage of a set of pixels

<u>Definition</u>:

Let $S = \{(x_i, y_i)\}$ be a set of pixels, its preimage S_p is:

$$S_p = \{(\alpha , \beta) \mid 0 \leq \alpha\, x_i + \beta - y_i < 1 \ ; \forall\, i\}$$

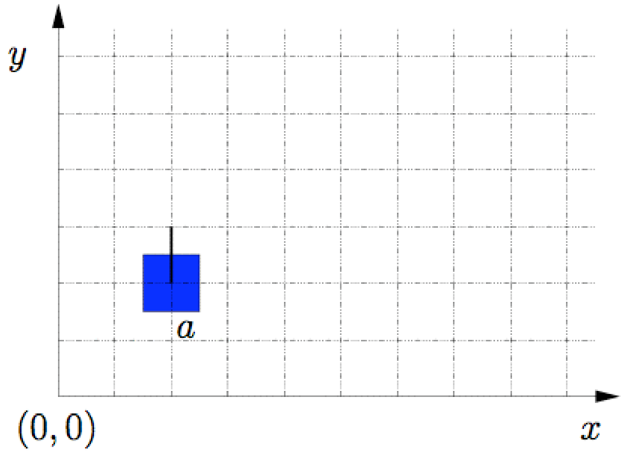
$$S_p = \cap_i \, B\, (x_i, y_i)$$

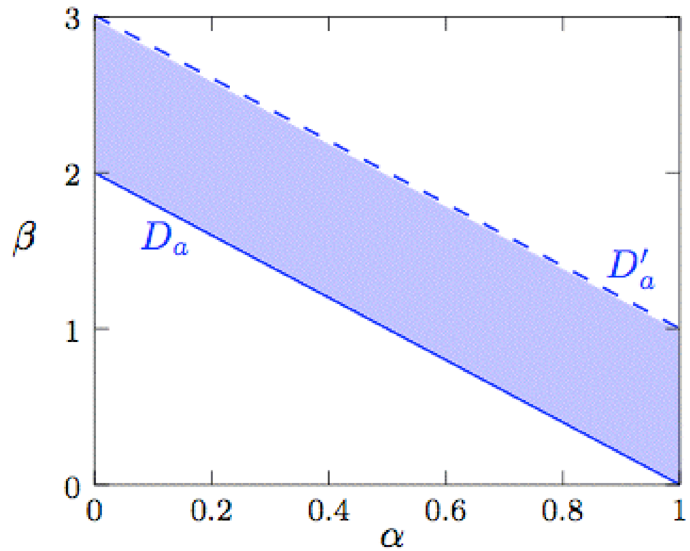
$(\alpha , \beta) \in S_p \;\Leftrightarrow\;$ the straight line with parameters (α , β) crosses the interval $[y_i, y_i{+}1[$ of each pixel.

Dorst & Smeulders, PAMI, 1984

McIlroy AT&T Tech Journal, 1985

Illustration of preimage



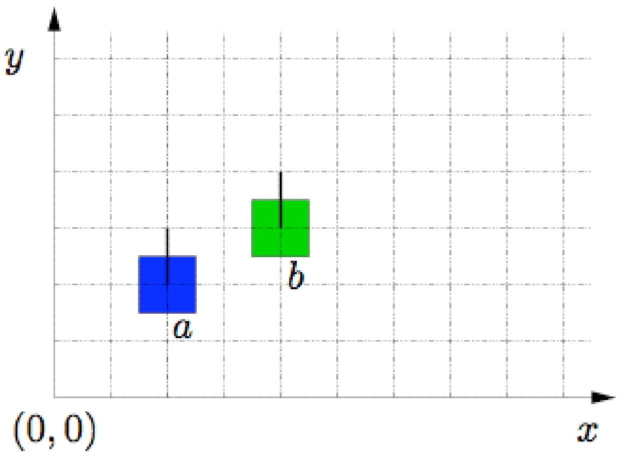


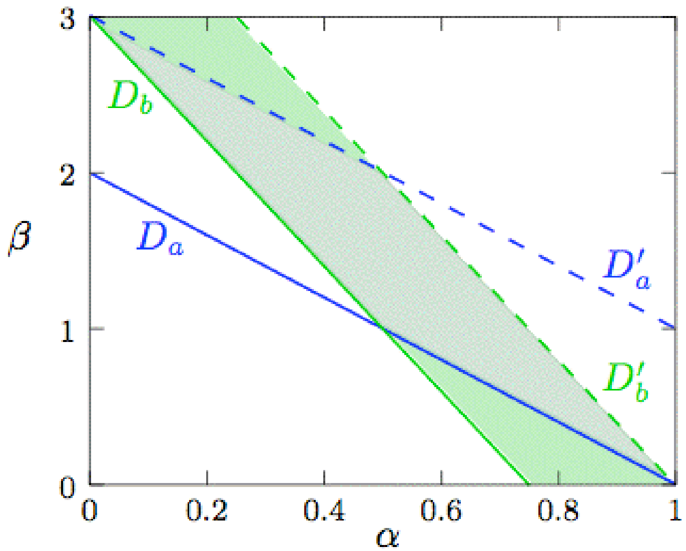
a=(2,2)

D_a : $2\,\alpha + \beta - 2 = 0$

D'_a : $2\,\alpha + \beta - 2 = 1$

Illustration of preimage



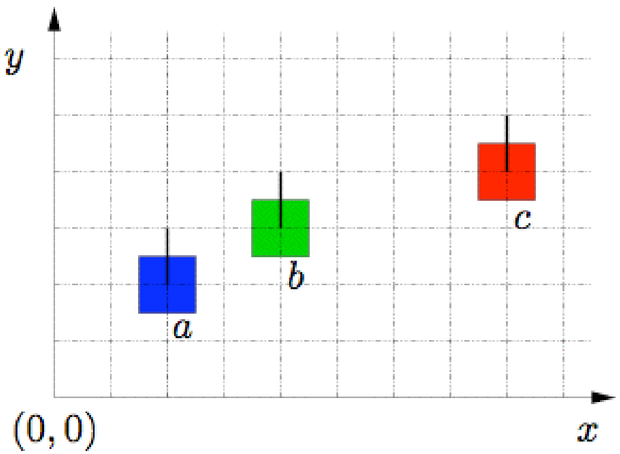


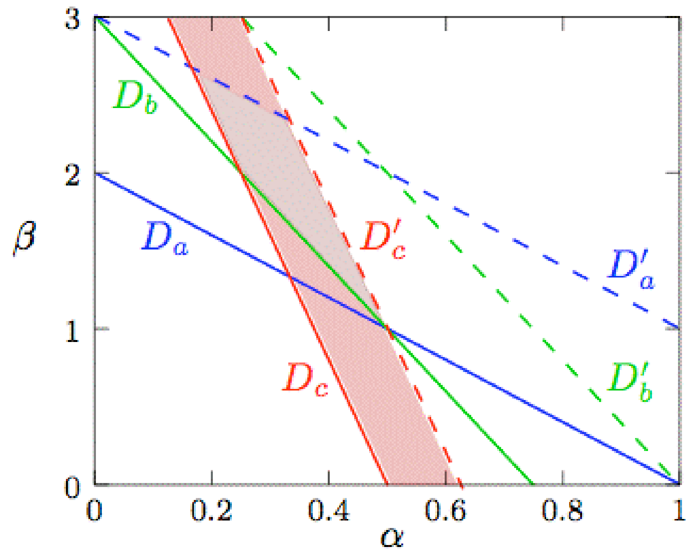
b=(4,3)

$D_b : 4\alpha + \beta - 3 = 0$

$D'_b : 4\alpha + \beta - 3 = 1$

Illustration of preimage



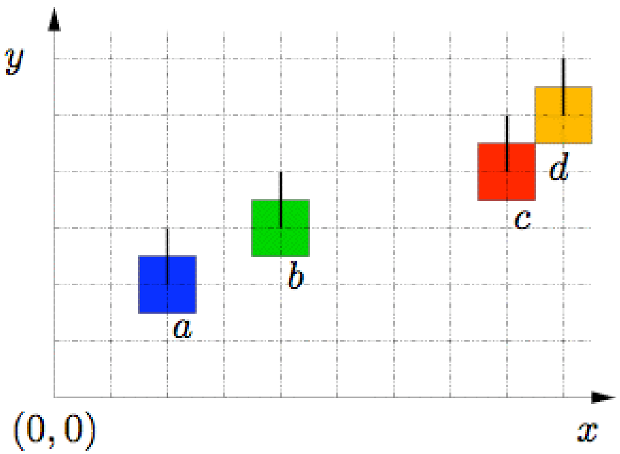


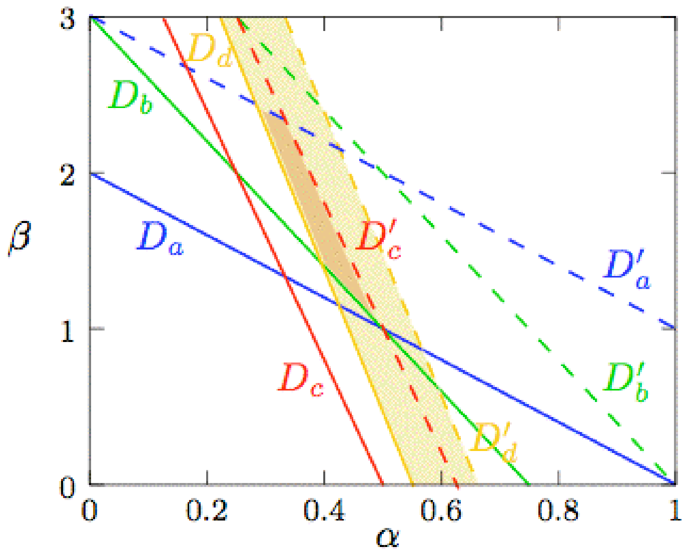
c=(8,4)

D_c : 8 α + β – 4 = 0

D'_c : 8 α + β – 4 = 1

Illustration of preimage



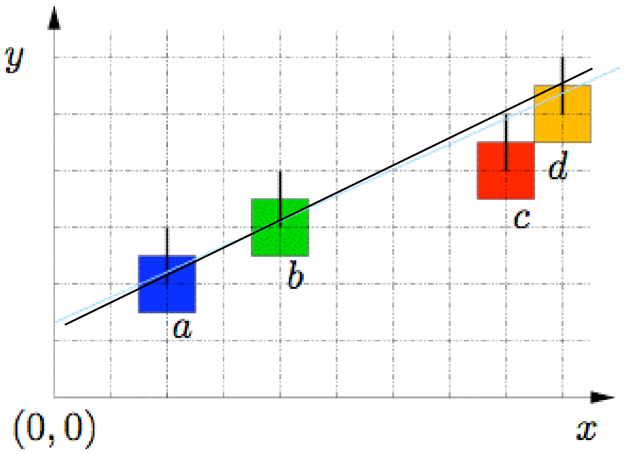


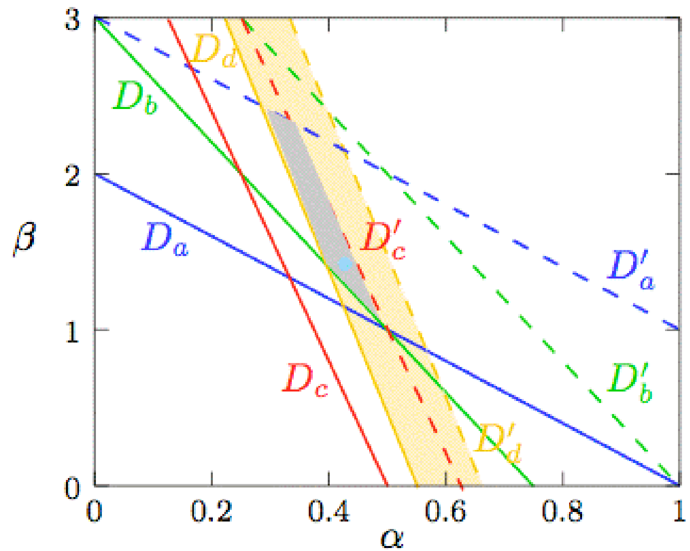
d=(9,5)

D_d : $9\alpha + \beta - 5 = 0$

D'_d : $9\alpha + \beta - 5 = 1$

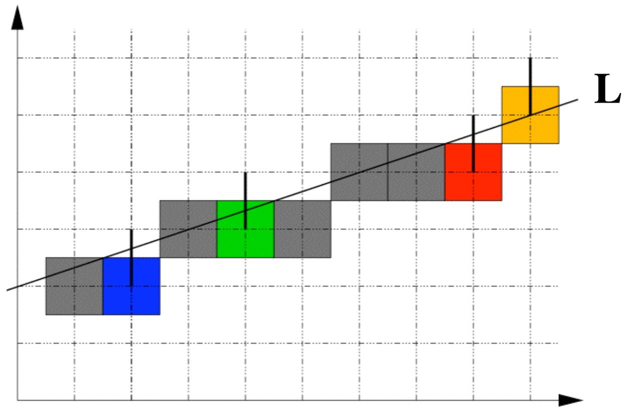
Illustration of preimage

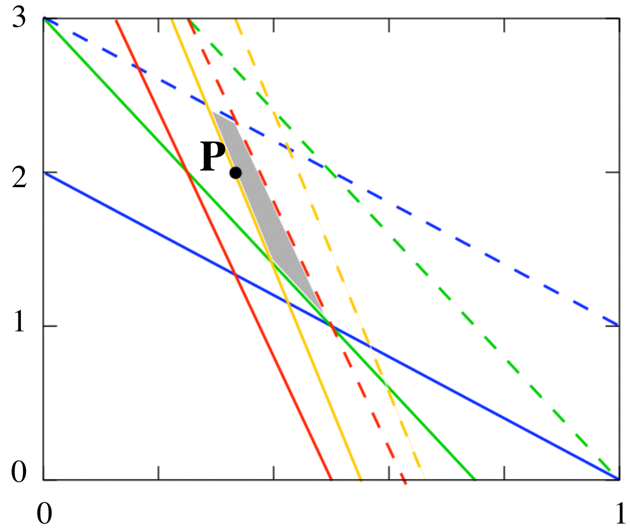




Set of solutions

Illustration of preimage



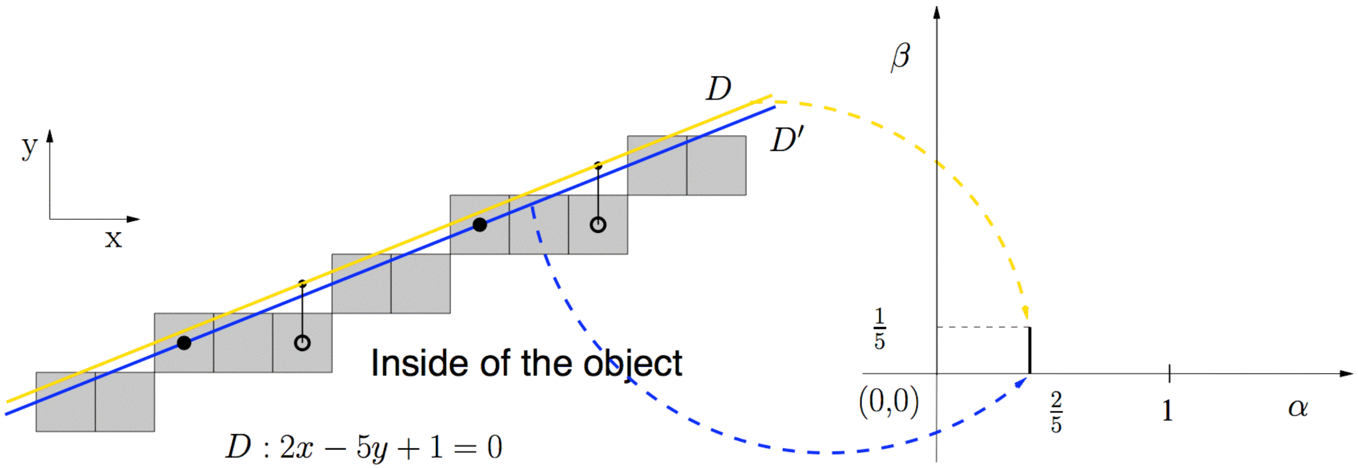


L : 1/3 x - y + 2 = 0

P (1/3, 2) involves the straight line L

Preimage of a straight line

In case of a straight line with parameters (α_0 , β_0) , the Preimage of the discretized infinite set (the naive line) is a vertical segment with extremities $(\alpha_0 , 0)$ and (α_0 , β_0).



$D : 2x - 5y + 1 = 0$

$D : 2x - 5y = 0$

Outline

1. Introduction
2. Hough transform and discrete geometry
3. Preimage of a pixel, a set of pixels, a connected straight line
4. Preimage and Farey sequences
5. Extension in 3D
6. Duality versus polarity
7. Polygon enclosure and inclusion problems
8. Application to the convex approximation by interior.
9. Conclusion

Farey series

Definition: The Farey series F_m of order m is the ascending series of irreducible fractions h/k between 0 and 1 whose denominators do not exceed m.

Property

- if h/k and h'/k' are two successive terms in F_m (with $h/k < h'/k'$), then $kh' - hk' = 1$.
- if h/k, h''/k'' and h'/k' are 3 successive terms in F_m (with $h/k < h''/k'' < h'/k'$), then $h''/k'' = (h+h')/(k+k')$. The fraction h''/k'' is called the mediant of h/k and h'/k'.
- F_{m+1} can be computed from F_m adding the mediant with denominator less or equal than $m + 1$ of each two successive fractions in F_m

F_1	{ 0/1,		1/1 }
F_2	{ 0/1,	1/2,	1/1 }
F_3	{ 0/1,	1/3, 1/2, 2/3,	1/1 }
F_4	{ 0/1,	1/4, 1/3, 2/5, 1/2, 3/5, 2/3, 3/4,	1/1 }

Hardy & Wright , 1989

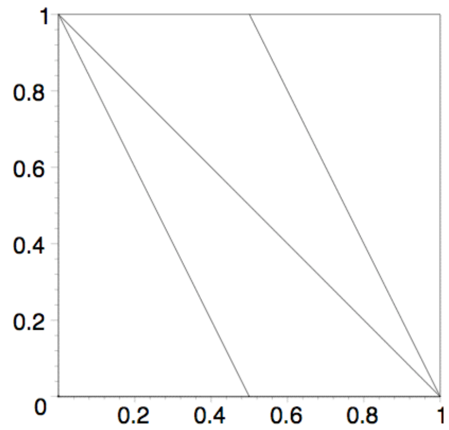
Farey Fan

Definition (Rays) $\boxed{R(x\ ,\ y\ ) = \{(\alpha\ ,\ \beta)\ |\ \beta = -\ \alpha\ x\ +y\ \}}$

According to the strip decomposition, for each point $(x\ ,\ y\ )$, we have the pair of parallel rays $R(x\ ,\ y\ )$ and $R(x\ ,\ y + 1)$ in the $(\alpha\ ,\ \beta)$ dual space.

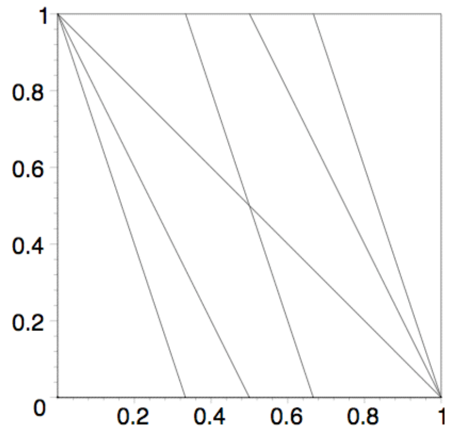
Farey Fans

If we focus on the domain [0, 1]x[0, 1[and segments in the first octant, rays $R(x\ ,\ y\ )$ with $0 \leq y \leq x \leq n$ define a diagram called the Farey fan of order n.



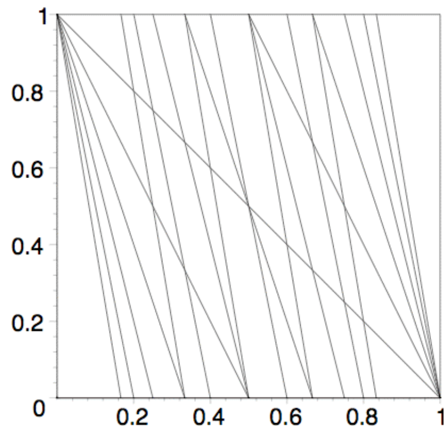
Farey serie of order 2

{ 0/1, 1/2, 1/1 }



Farey serie of order 3

{ 0/1, 1/3, 1/2, 2/3, 1/1 }



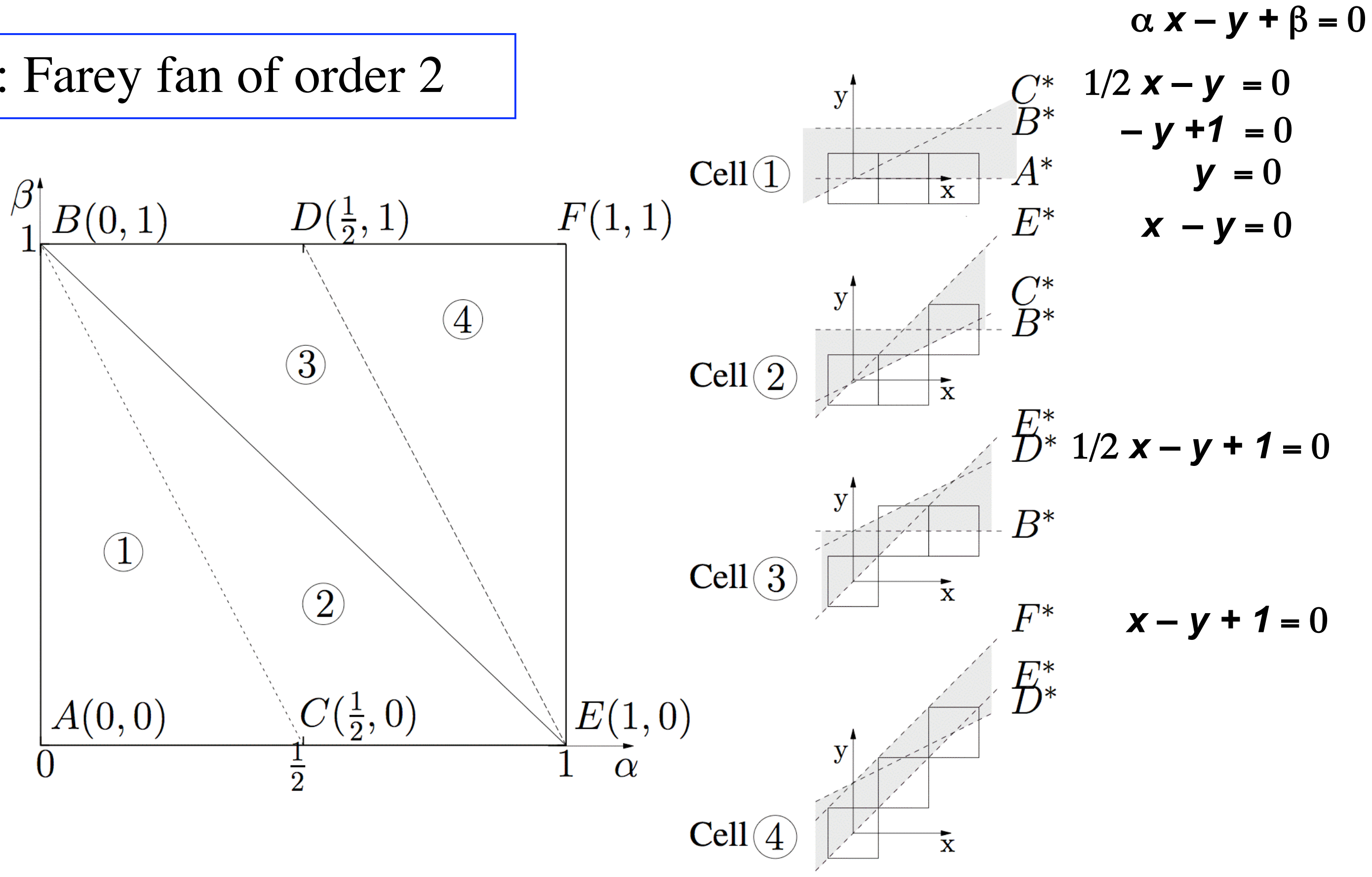
Farey serie of order 6

{ 0/1, 1/6, 1/5, 1/4, 1/3, 2/5, 1/2, 3/5, 2/3, 3/4, 4/5, 5/6, 1/1 }

Farey fan and Digital Straight line

Farey fan of order n represents all the Digital Straight line of length n+1

Example: Farey fan of order 2

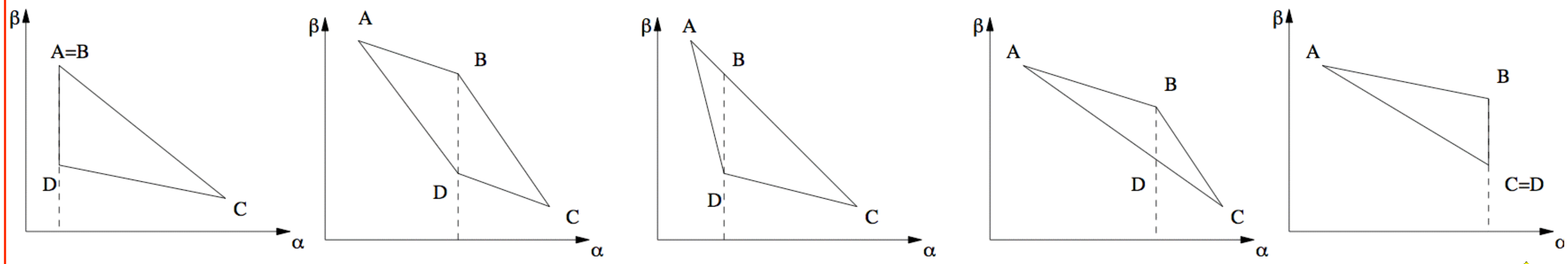


Preimage of a naive segment

<u>Property</u>:

Let S be a set of (N+1) 8-connected pixels and x_0 be the minimum abscissa of this set of pixels. Then the preimage S_p of S has the following properties:

1. S_p is a convex polygon with at most four vertices;
2. Two consecutive abscissa of the vertices are consecutive terms in the Farey series of order max $(x_0, N-x_0)$. Moreover, for a given abscissa equal to p/q, the corresponding ordinate is a multiple of $1/q$;
3. If this polygon has four vertices, then two out of the four vertices have the same abscissa.



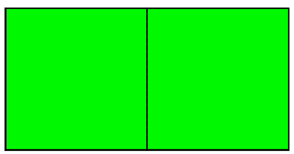
Recognition of a digital straight line based on preimage

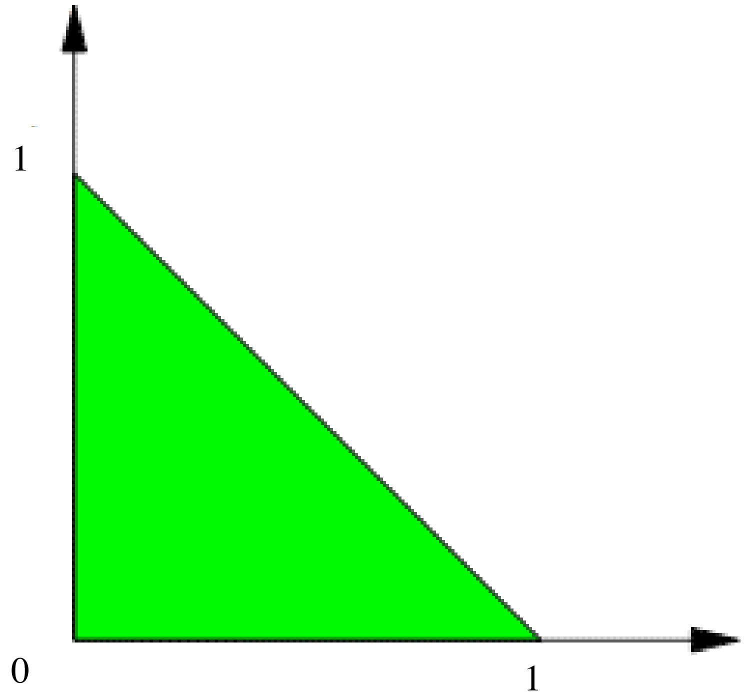
Let S be a set of pixels.

Property: S is a piece of digital straight line iff

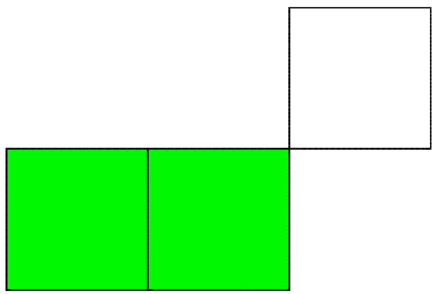
the preimage S_p of S is not empty.

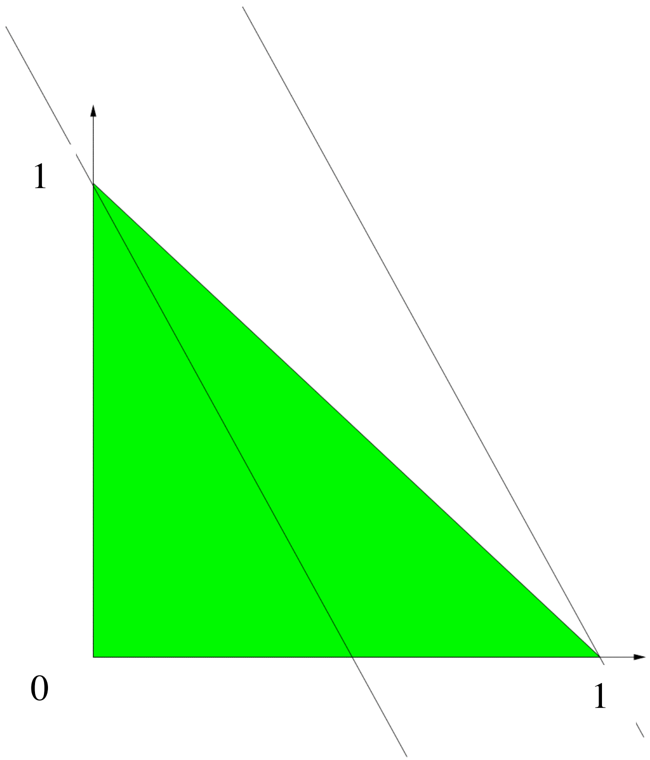
Illustration





Step: Initialization
S_p coordinates: {(0, 1), (0, 0), (1, 0), (0, 1)
slopes: {(_ _, (0, 0), (1, 1), (_ _}

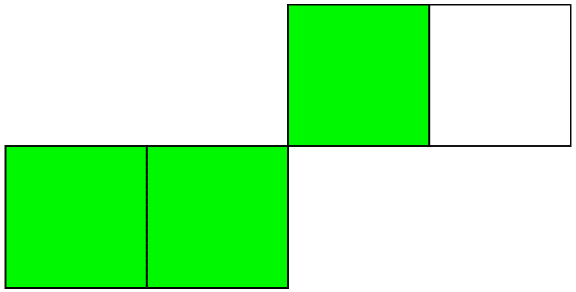


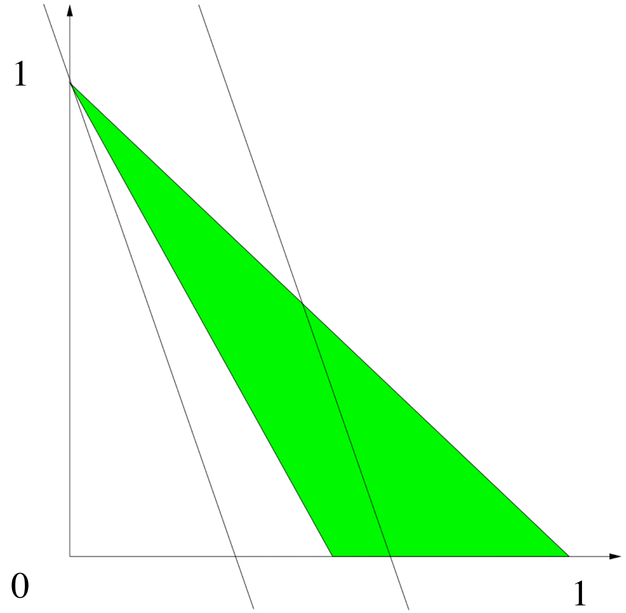


Step: $D \cap Llow \# \varnothing$

S_p coordinates: {(0, 1), (1/2 , 0), (1, 0), (1/2 , 1/2)}

slopes: {(2, 1), (0, 0), (1, 1), (1, 1)}

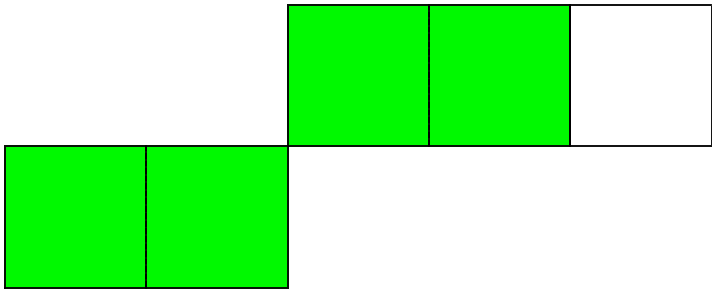


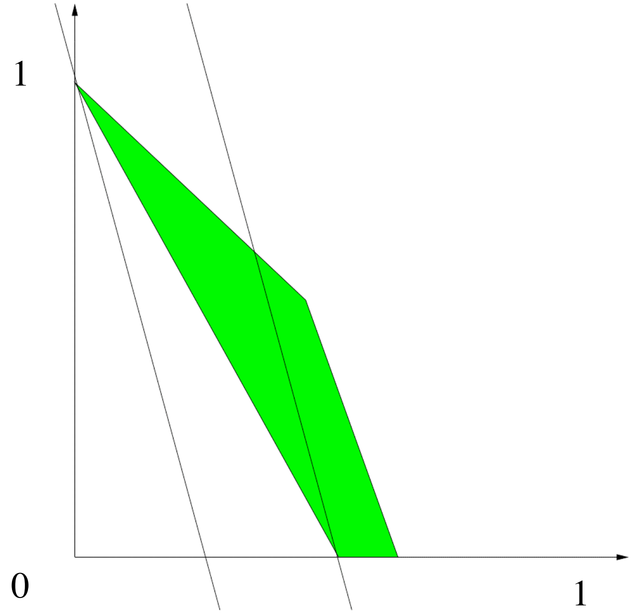


Step: $D \cap Lhigh \# \emptyset$

S_p coordinates: {(0, 1), (1/2 , 0), (2/3 , 0), (1/2 , 1/2)}

slopes: {(2, 1), (0, 0), (3, 1), (1, 1)}

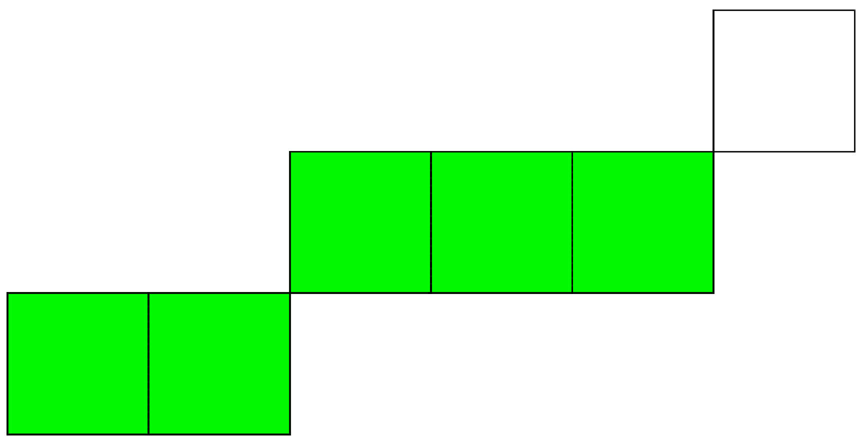


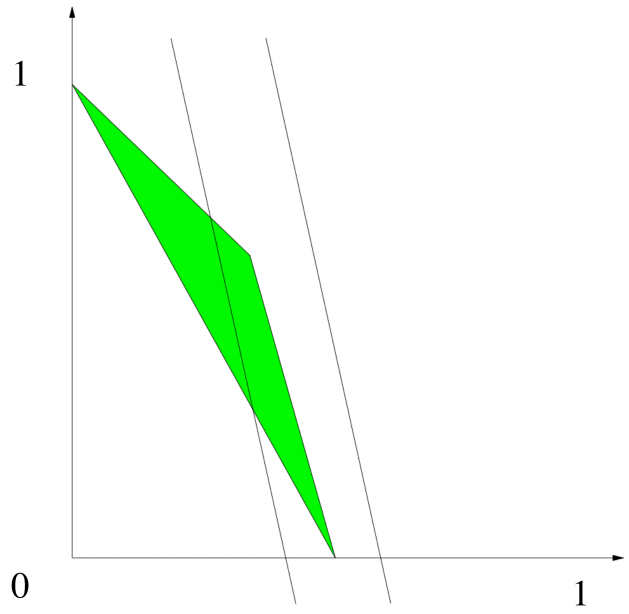


Step: $B \cap Lhigh \# \emptyset$

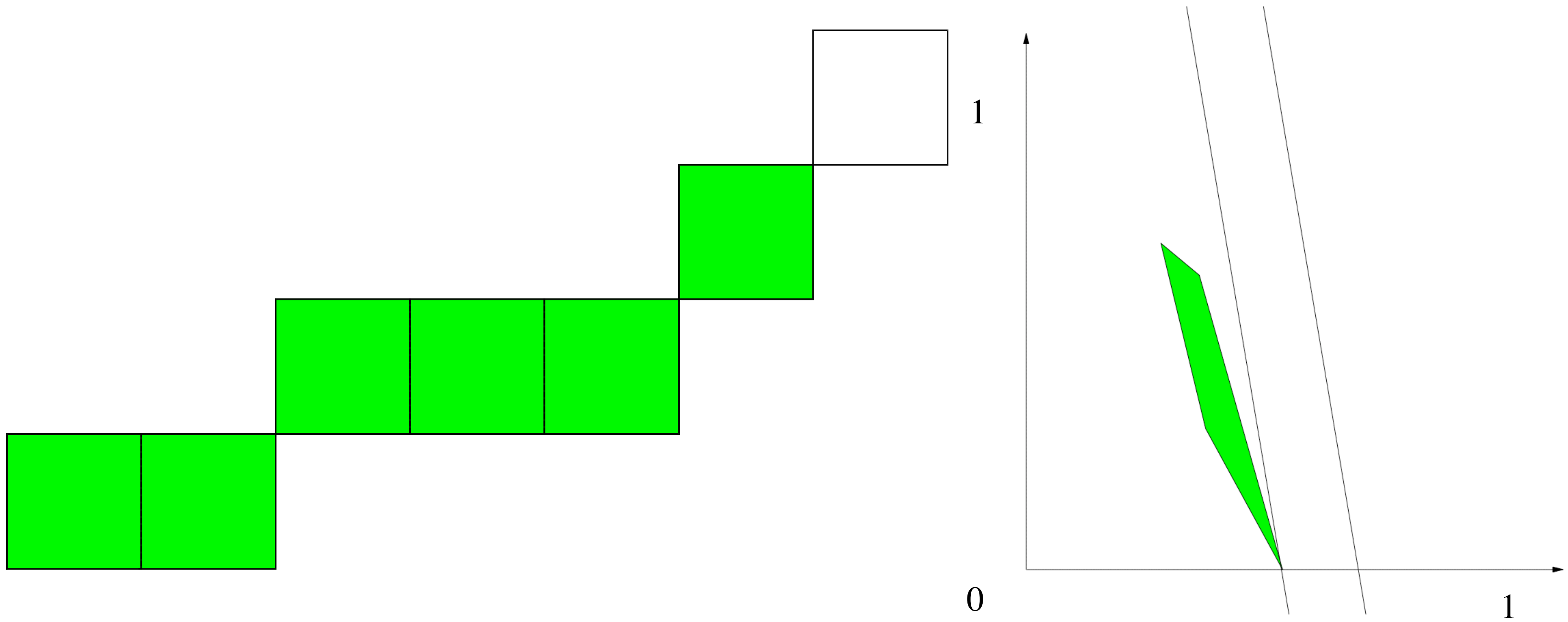
S_p coordinates: {(0, 1), (1/3 , 1/3), (1/2 , 0), (1/3 , 2/3)}

slopes: {(2, 1), (0, 0), (4, 1), (1, 1)}





Step: $B \cap Llow$ # $\emptyset$
S_p coordinates: {(1/4 , 3/4), (1/3 , 1/3), (1/2 , 0), (1/3 , 2/3)}
slopes: {(5, 2), (0, 0), (4, 1), (1, 1)}



Step: $B \notin \mathrm{B}(p)$ or $D \notin \mathrm{B}(p)$
S_p coordinates: $\emptyset$
slopes: $\emptyset$

Outline

1. Introduction
2. Hough transform and discrete geometry
3. Preimage of a pixel, a set of pixels, a connected straight line
4. Preimage and Farey sequences
5. Extension in 3D
6. Duality versus polarity
7. Polygon enclosure and inclusion problems
8. Application to the convex approximation by interior.
9. Conclusion

Digital Plane: Parametric Equation

$0 \leq ax + by + cz + r < \omega$

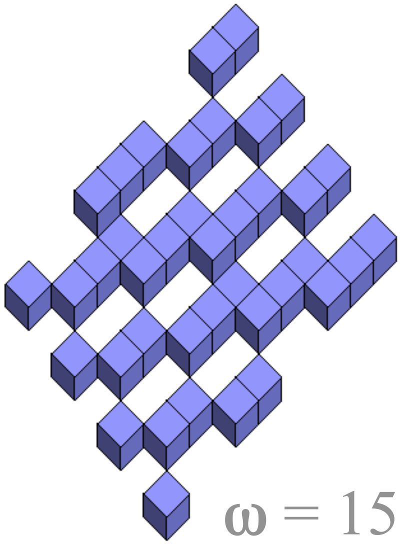
ω: thickness

(a,b,c) : normal vector

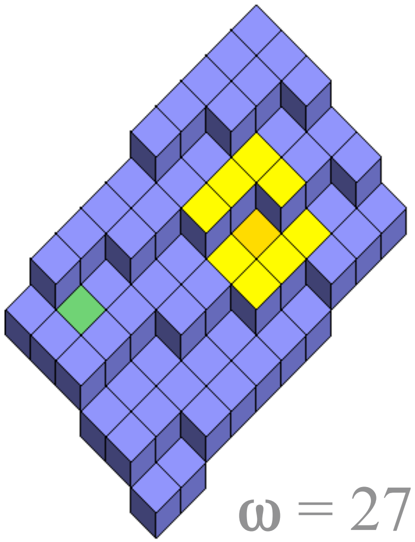
r : translation parameter

Example (a,b,c,r):

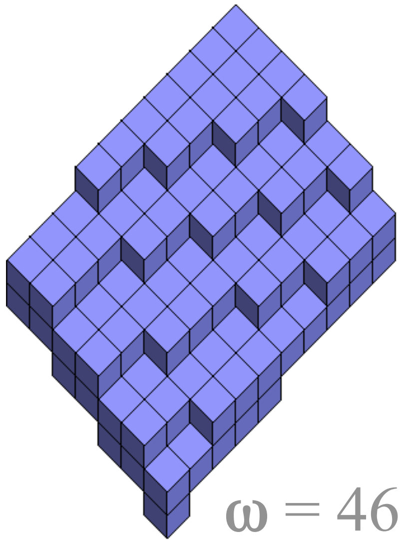
P = (6,13,27,0)



$\omega = 15$



$\omega = 27$



$\omega = 46$

Digital Naive Plane

$0 \leq ax + by + cz + r < \omega$

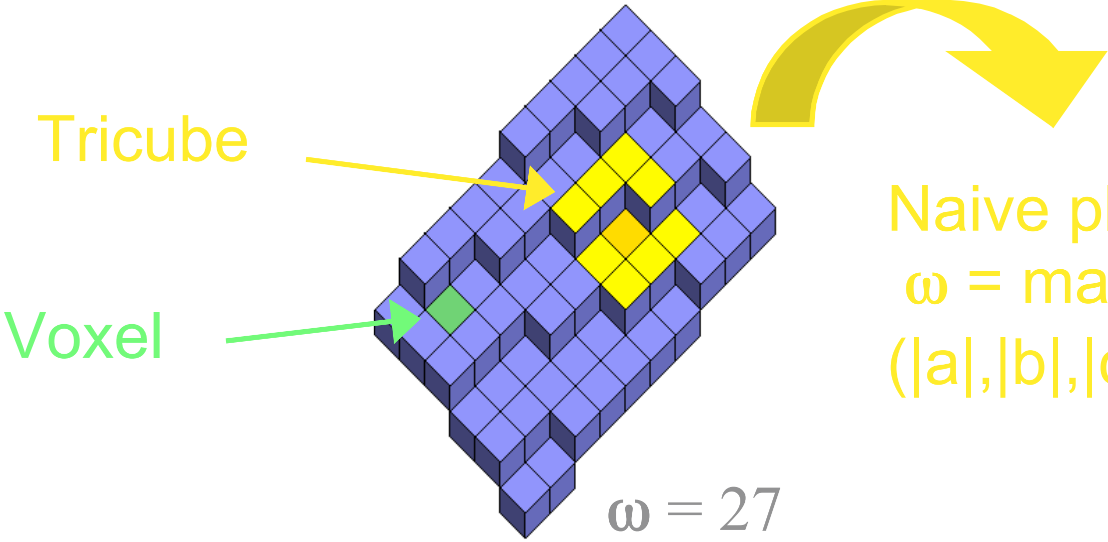
ω: thickness

(a,b,c) : normal vector

r : translation parameter

Example (a,b,c,r):

P = (6,13,27,0)



Naive plane :
$\omega = \max(|a|,|b|,|c|)$

Extension in 3D in the dual space

Let $P(x_0, y_0, z_0)$ be a voxel and $L : z= \alpha\, x + \beta y + \gamma$ a straight line

$$P \in OBQ(L) \Leftrightarrow z_0 = \text{Floor}\,(\alpha\, x_0 + \beta\, y_0 + \gamma)$$

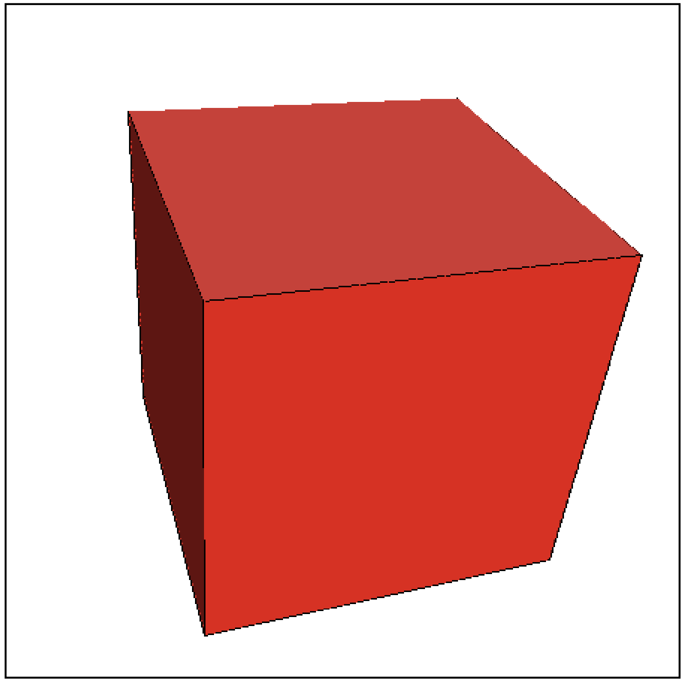
$$0 \leq \alpha\, x_0 + \beta\, y_0 + \gamma - z_0 < 1$$

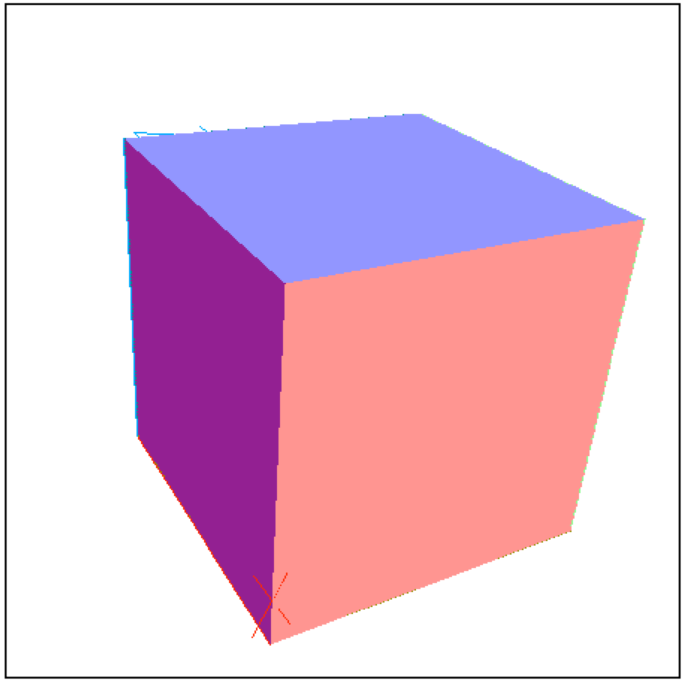
Strip associated to a voxel

(α, β, γ) such that:

$$B\,(x_0, y_0, z_0) = \{\ 0 \leq \alpha\, x_0 + \beta\, y_0 + \gamma - z_0 \quad (D)$$
$$\text{and} \quad \alpha\, x_0 + \beta\, y_0 + \gamma - z_0 < 1 \quad (D')\ \}$$

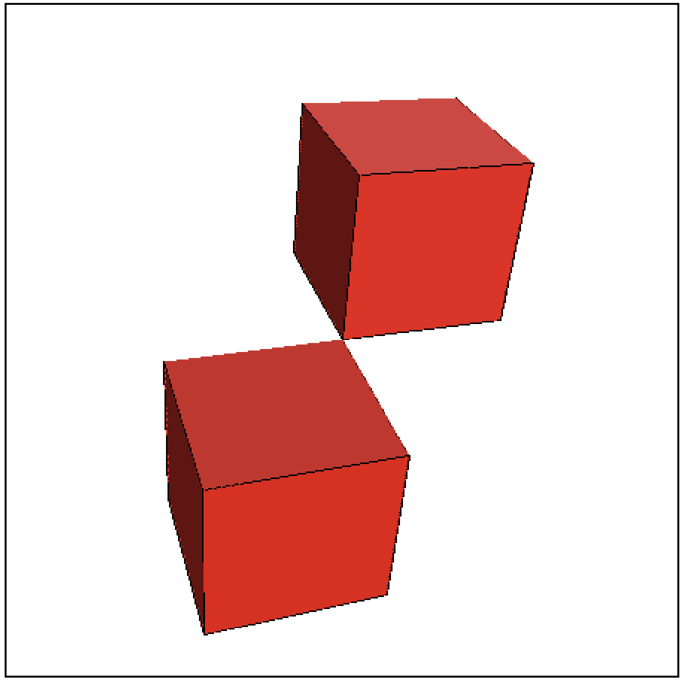
Recognition example

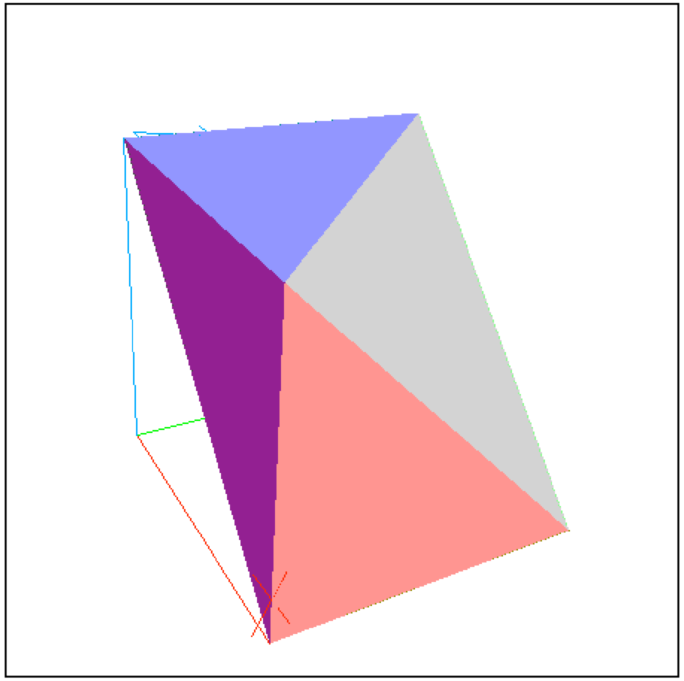




- **(0, 0, 0)**
- (1, 1, 1)
- (2, 1, 1)
- (2, 2, 1)
- (3, 2, 1)

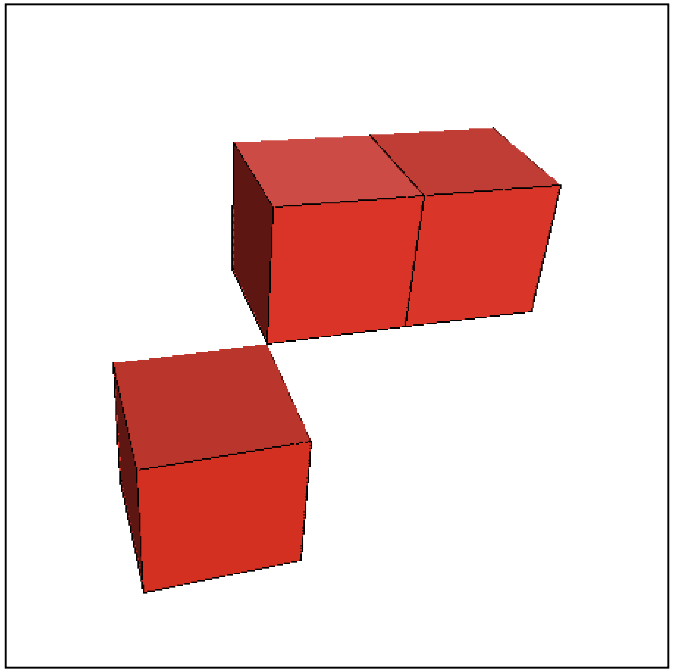
Recognition example

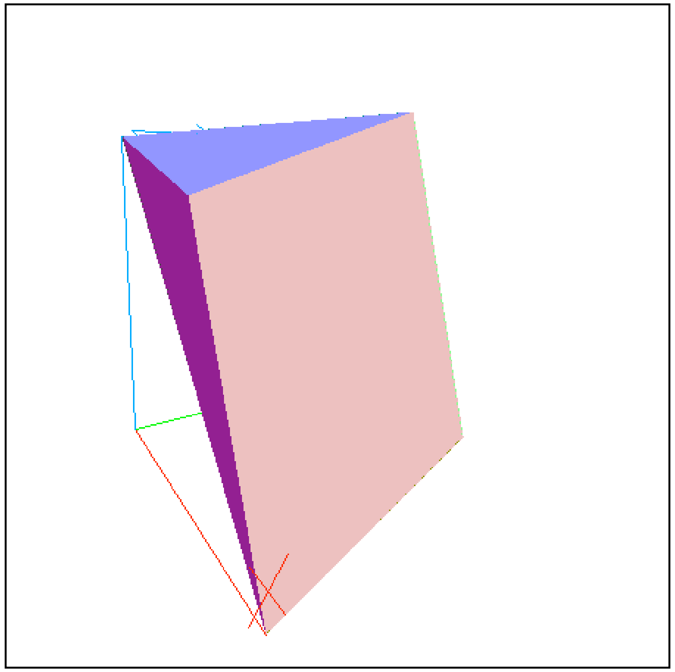




- **(0, 0, 0)**
- **(1, 1, 1)**
- (2, 1, 1)
- (2, 2, 1)
- (3, 2, 1)

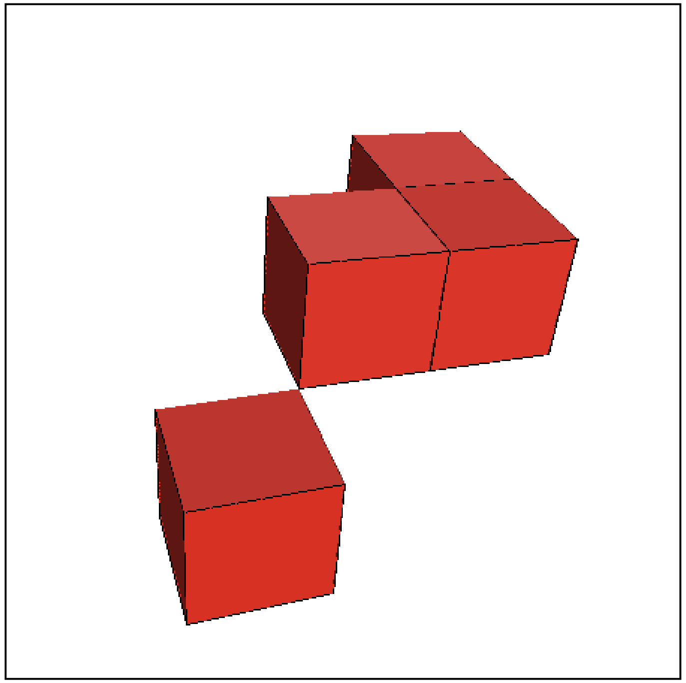
Recognition example

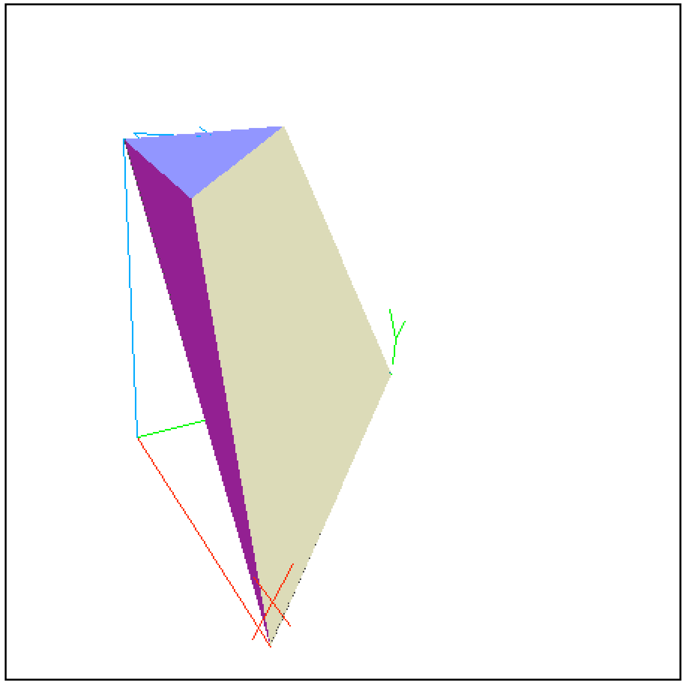




- **(0, 0, 0)**
- **(1, 1, 1)**
- **(2, 1, 1)**
- (2, 2, 1)
- (3, 2, 1)

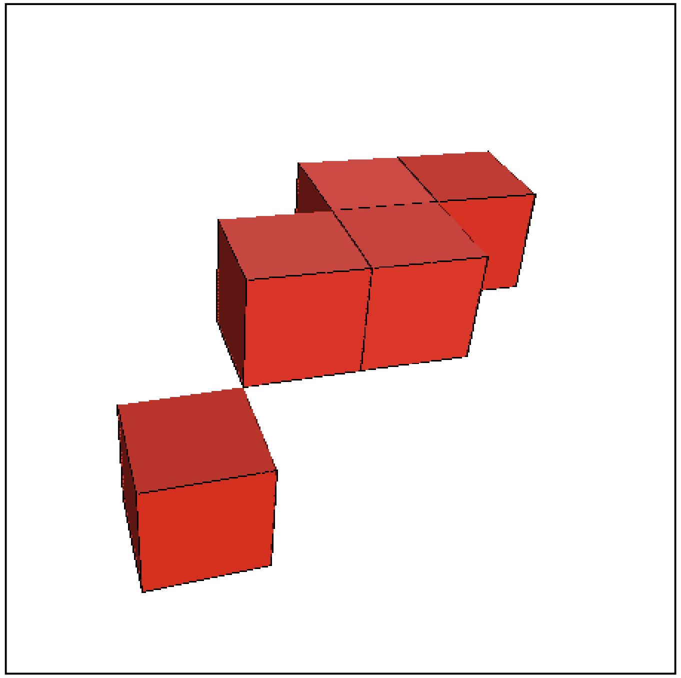
Recognition example

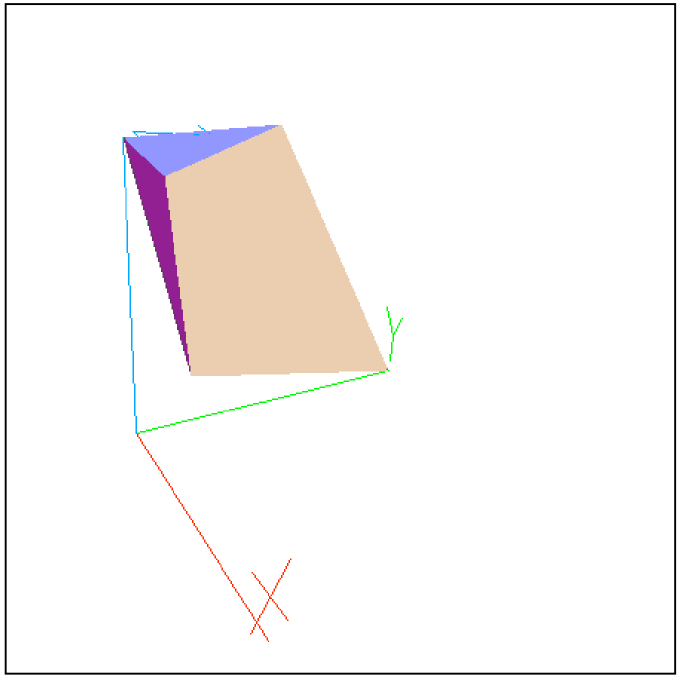




- (0, 0, 0)
- (1, 1, 1)
- (2, 1, 1)
- (2, 2, 1)
- (3, 2, 1)

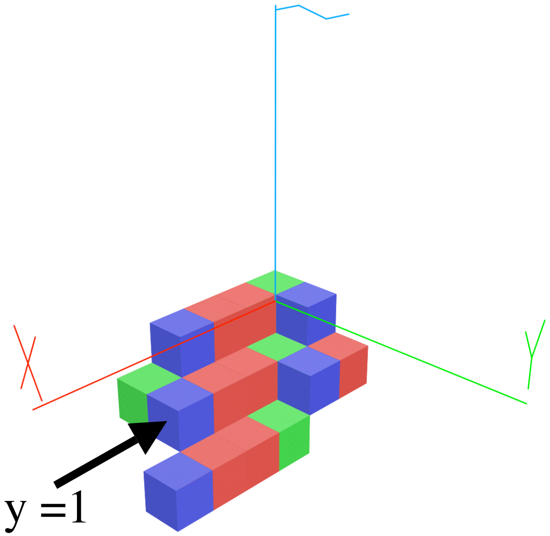
Recognition example

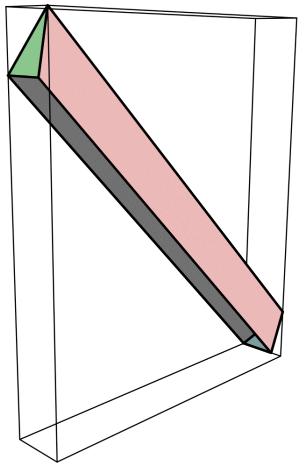




- **(0, 0, 0)**
- **(1, 1, 1)**
- **(2, 1, 1)**
- **(2, 2, 1)**
- **(3, 2, 1)**

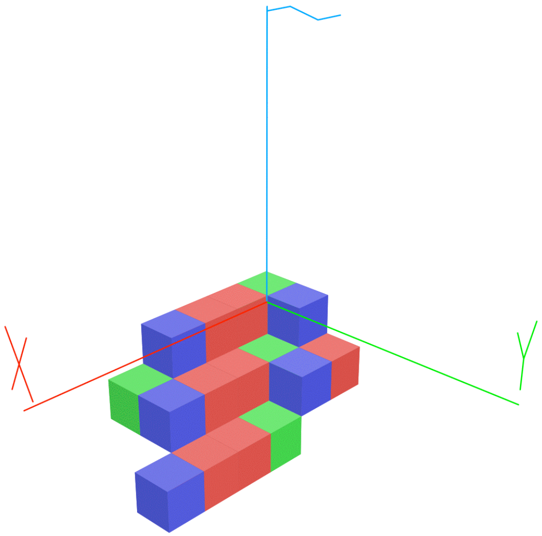
Preimage of a 3D straight line

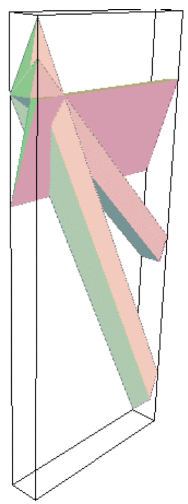


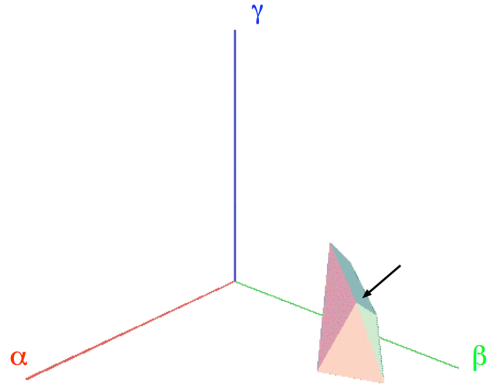


Preimage of the 3D digital straight line defined by y =1 in the plane P(1,3,4,0)

Preimage of a part of 3D naive plane



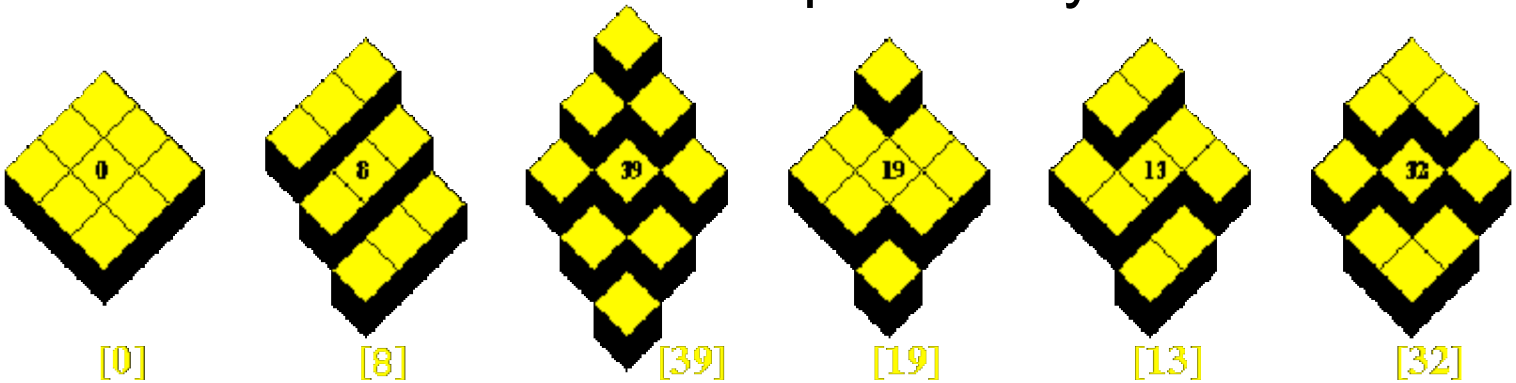




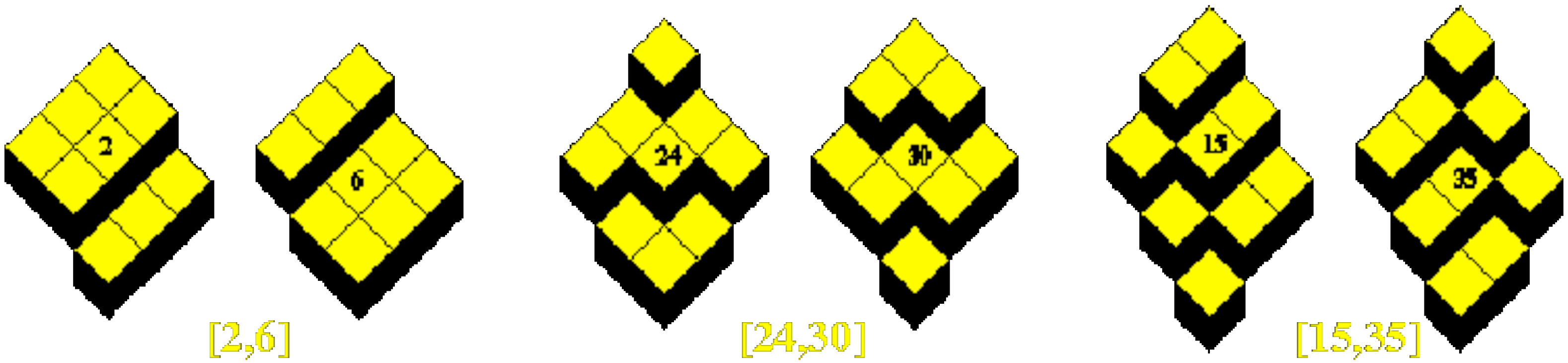
A piece of plane P(1,3,4,0)	Preimages of 3D digital straight lines in parameter space (a, b, g)	The preimage of the piece of plane is the intersection of the digital lines preimages.

Tricubes (1/2)

40 tricubes = 6 neutral + 17 pairs of symetrical

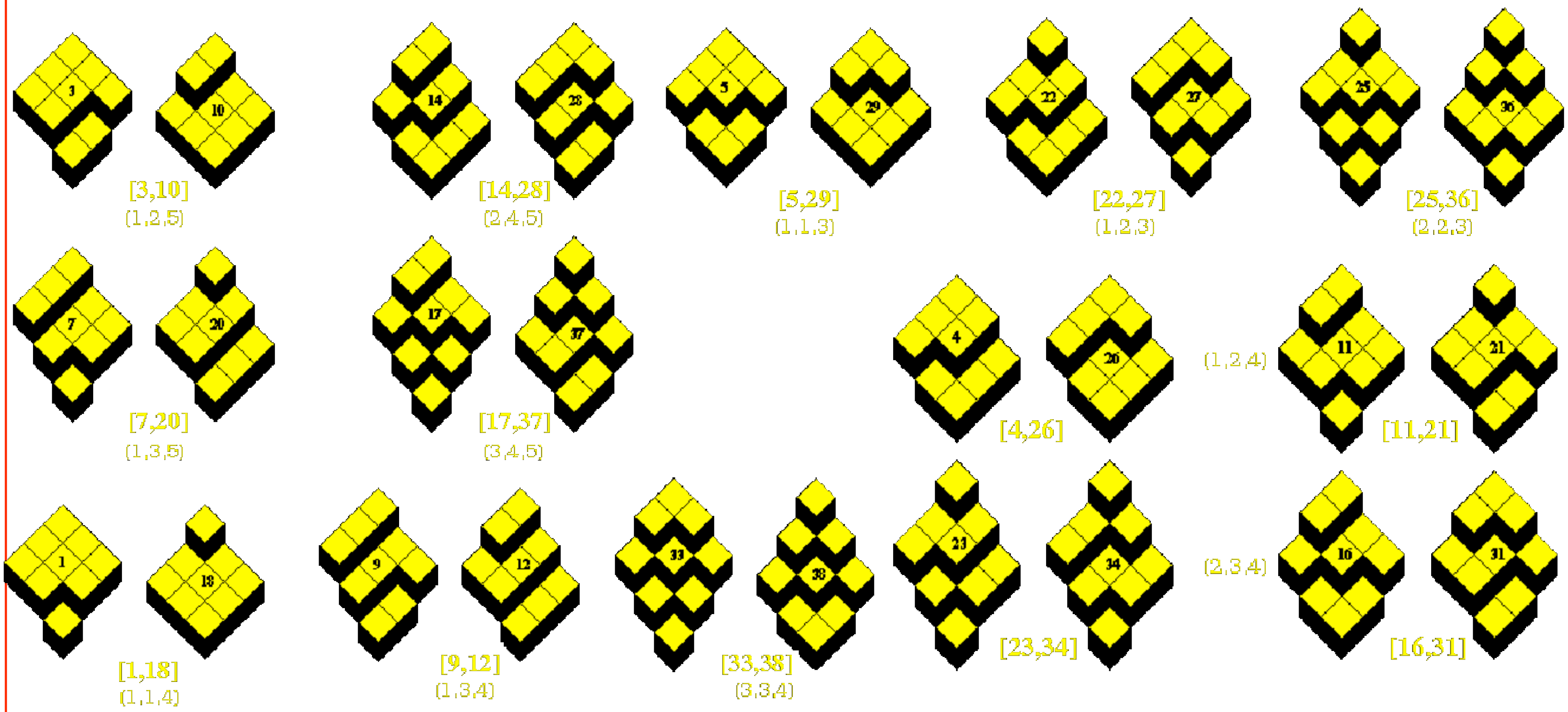


Identifier	[0]	[8]	[39]	[19]	[13]	[32]
Normal vector	(0,0,1)	(0,1,1)	(1,1,1)	(1,1,3)	(1,2,3)	(2,2,3)



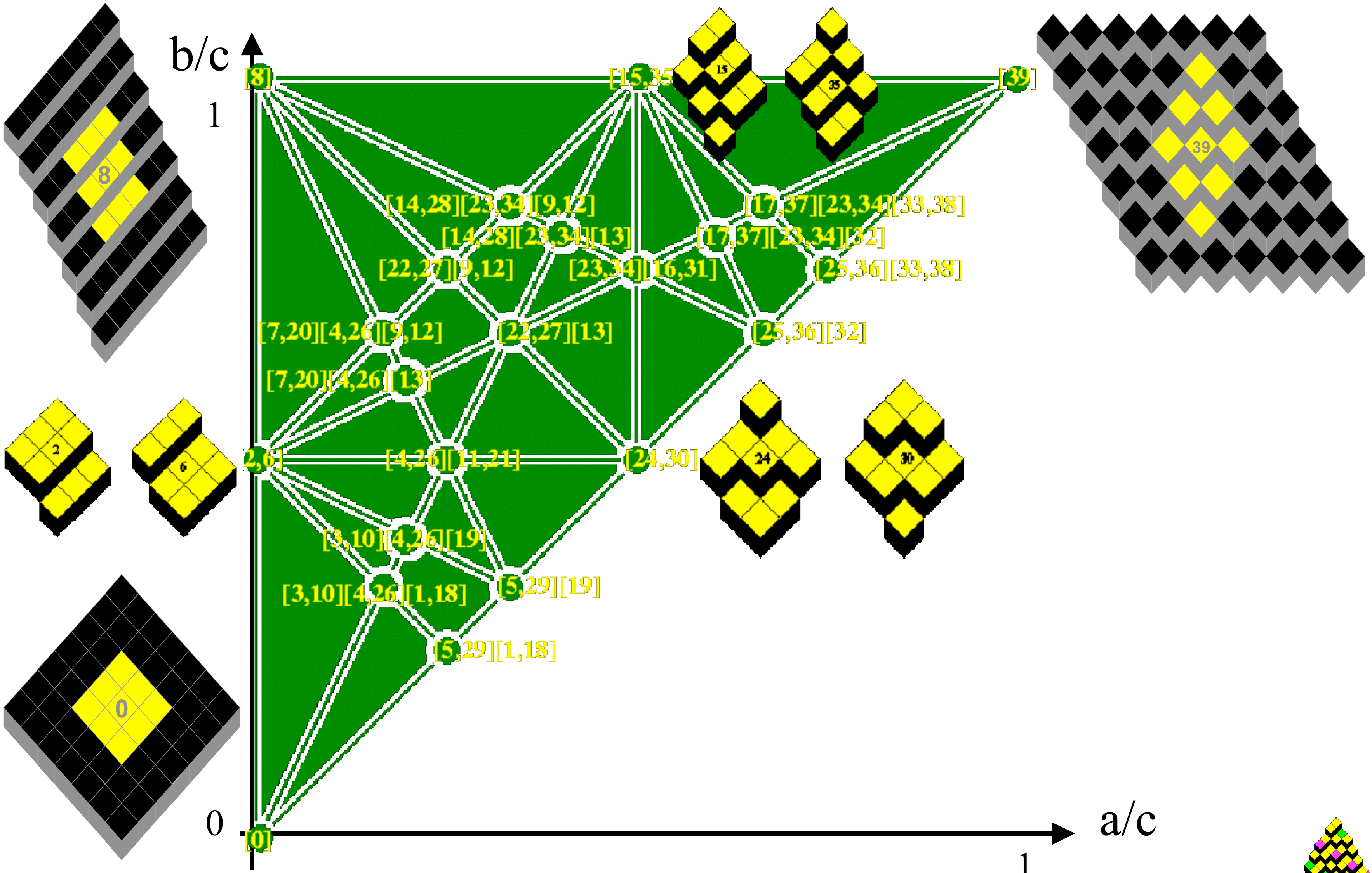
[2,6]	[24,30]	[15,35]
(0,1,2)	(1,1,2)	(1,2,2)

Tricubes (2/2)



[3,10]
(1,2,5)
[14,28]
(2,4,5)
[5,29]
(1,1,3)
[22,27]
(1,2,3)
[25,36]
(2,2,3)
[7,20]
(1,3,5)
[17,37]
(3,4,5)
[4,26]
(1,2,4)
[11,21]
[1,18]
(1,1,4)
[9,12]
(1,3,4)
[33,38]
(3,3,4)
[23,34]
(2,3,4)
[16,31]

Farey fan associated to tricubes



Tricubes - Properties

1. A naive plane contains no more than **9** different types of tricubes.

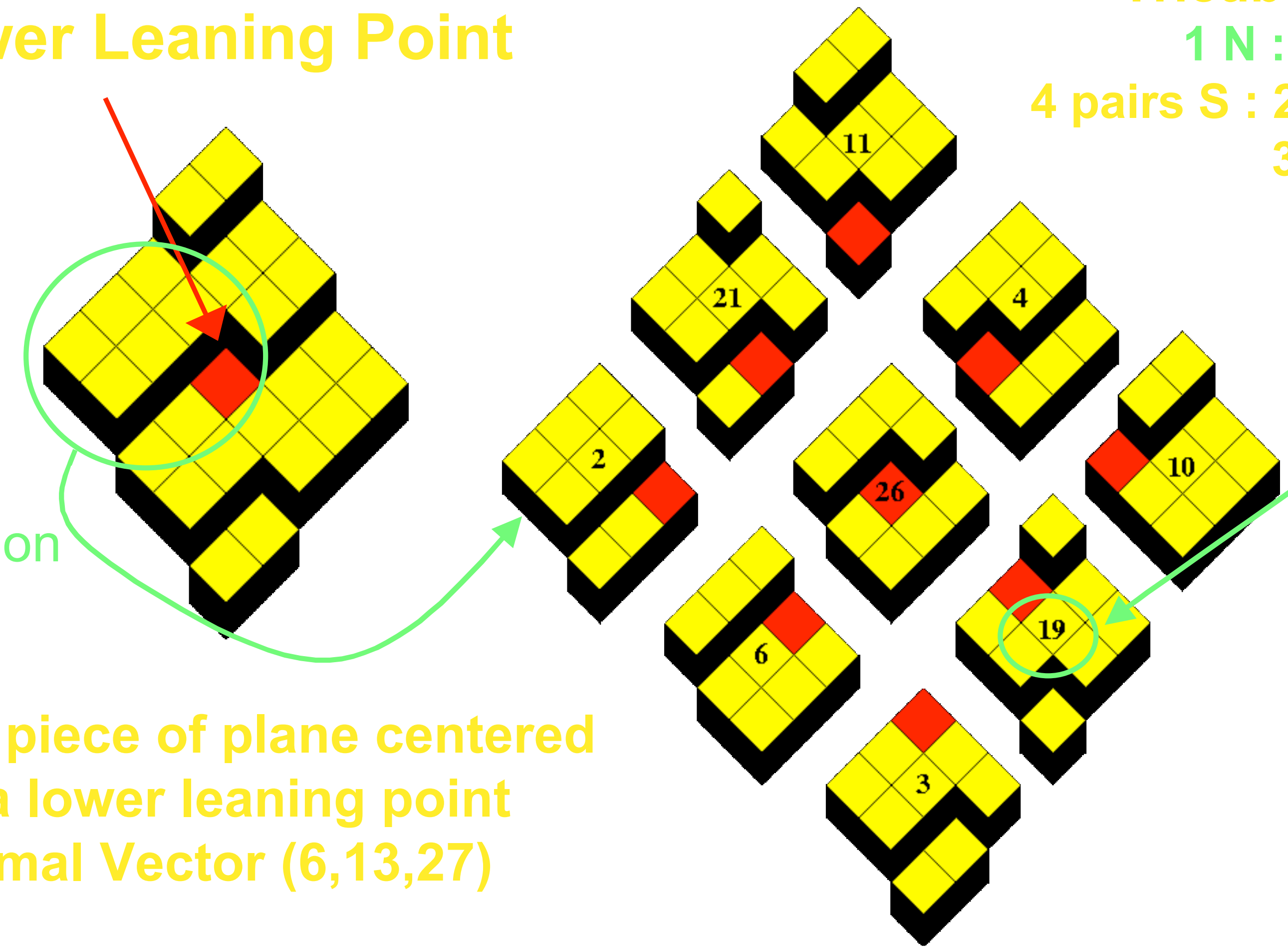
2. A **5x5** piece of naive plane centered on a leaning point contains **all** the tricubes used in the plane.

3. If the number of tricubes is :
   - even : all the tricubes are present by pairs
   - odd : one tricube is neutral and others are present by pairs

Tricubes - Properties

Lower Leaning Point

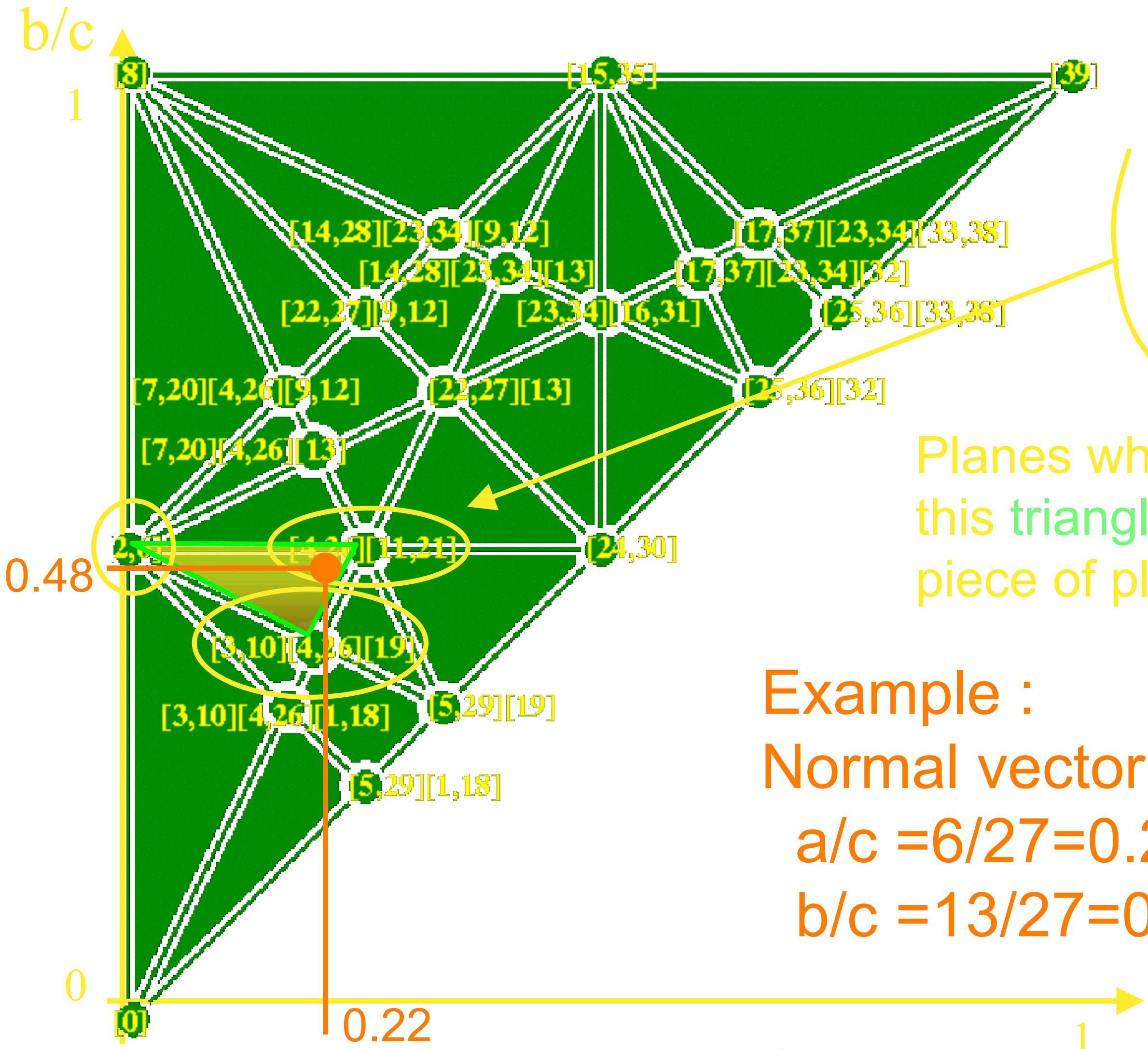
Exactly 9 types of Tricubes :
1 N : 19
4 pairs S : 2&6; 11&21
3&10; 4&26



Tricubes Decomposition

5x5 piece of plane centered on a lower leaning point
Normal Vector (6,13,27)

Farey diagram associated to tricubes

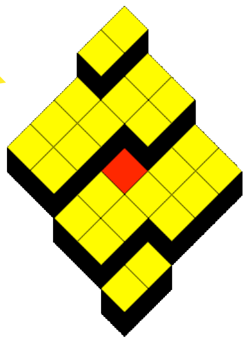


Exactly 9 types of Tricubes :

1 N : 19

4 pairs S : 2&6; 11&21
3&10; 4&26

Planes which parameters are on this triangle, contain the initial piece of plane



Example :

Normal vector (6,13,27)

a/c =6/27=0.22

b/c =13/27=0.48

Outline

1. Introduction
2. Hough transform and discrete geometry
3. Preimage of a pixel, a set of pixels, a connected straight line
4. Preimage and Farey sequences
5. Extension in 3D
6. Duality versus polarity
7. Polygon enclosure and inclusion problems
8. Application to the convex approximation by interior.
9. Conclusion

Homogeneous representation

The homogeneous representation of a point P = $(x_1, x_2, \ldots, x_d)$ in the d-dimensional Euclidean space is the point $(x_1, x_2, \ldots, x_d, 1)$ in the projective space.

For any non-zero scalar λ, the homogeneous points $(\lambda x_1, \lambda x_2, \ldots, \lambda x_d, \lambda)$ represent the same point in the Euclidean space.

This representation framework is convenient to obtain a matrix representation of both affine transformations and duality mapping.

Polarity

To define the polarity we consider the transformation:

$$\alpha = \beta \boldsymbol{B}$$

with α and β vectors in (d+1) homogeneous coordinate

and $\boldsymbol{B}$ a (d+1) x (d+1) matrix.

In 2D this transformation maps points to lines and lines to points.

This class of transformations preserves the incidence:

$$\alpha \in \beta \text{ , then } \beta \boldsymbol{B} \in \alpha \boldsymbol{B}$$

The matrix $\boldsymbol{B}$ is such that $\alpha \boldsymbol{B} \alpha^T = 0$ which corresponds to equation of a conic in homogeneous coordinates.

Duality and polarity in 2D

Let us focus on polarity defined by the matrix:

$$\boldsymbol{B} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{pmatrix}$$

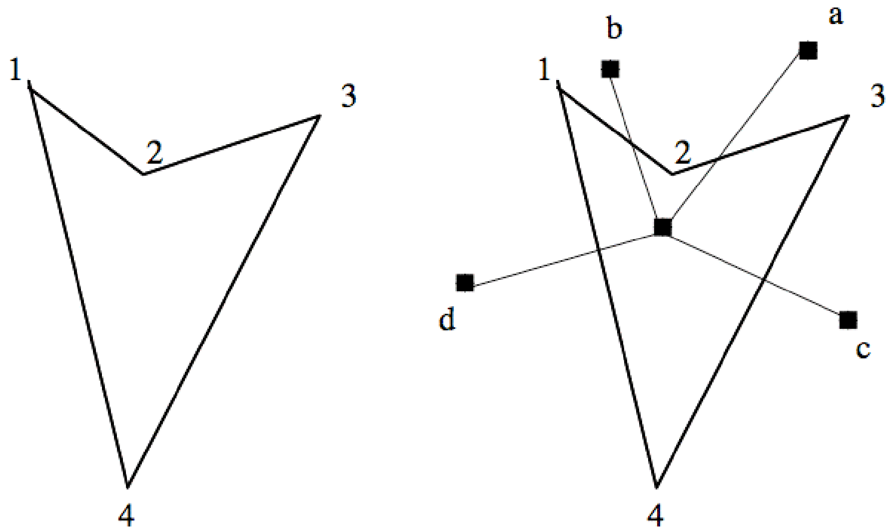
A point $(x_1, x_2, 1)$ is mapped to the dual line $x_1x + x_2y - z = 0$

The conic defined by this mapping is a unit circle in 2D.

Given a point (or a line) $\boldsymbol{\alpha}$, we have:

$$\textit{distance}\ (\boldsymbol{\alpha}, O)\ .\ \textit{distance}\ (\mathrm{dual}(\boldsymbol{\alpha}), O) = 1$$

Polarity

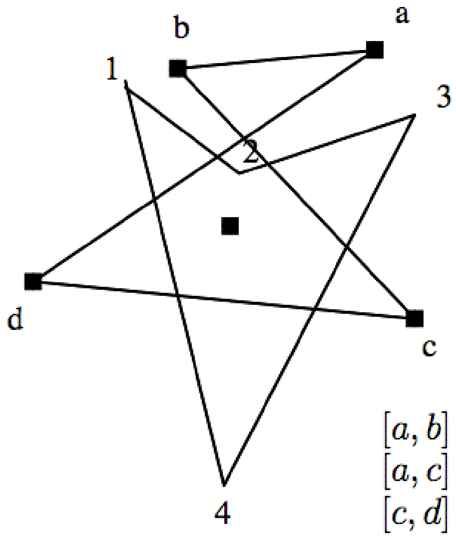


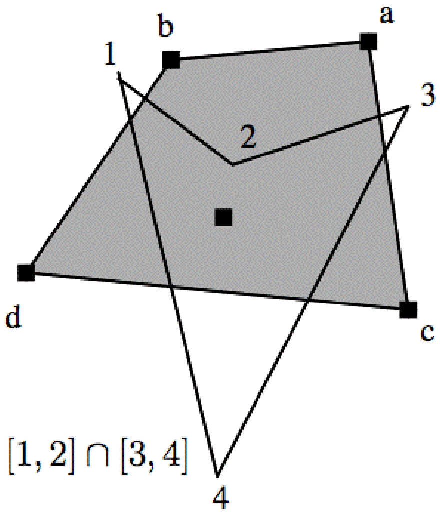
$$[1,2] \rightarrow a$$
$$[2,3] \rightarrow b$$
$$[3,4] \rightarrow c$$
$$[4,1] \rightarrow d$$

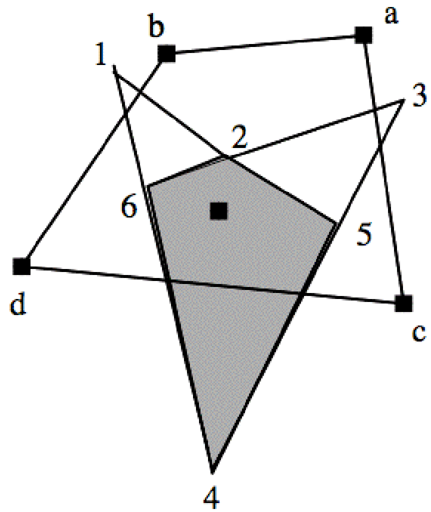
$$[a,b] \rightarrow 2$$
$$[b,c] \rightarrow 3$$
$$[c,d] \rightarrow 4$$
$$[d,a] \rightarrow 1$$

Union and Intersection are “ dual ”

Polarity - Convexity - Kernel







$[a, b] \to 2$
$[a, c] \to 5 = [1, 2] \cap [3, 4]$
$[c, d] \to 4$
$[d, b] \to 6 = [4, 1] \cap [2, 3]$

$dual[1, 2, 3, 4] = [a, b, c, d]$

$dual[a, b, d, c] = [2, 6, 4, 5] = Kern[1, 2, 3, 4]$

The convex hull of the polar figure is the polar of the kernel.

Outline

1. Introduction
2. Hough transform and discrete geometry
3. Preimage of a pixel, a set of pixels, a connected straight line
4. Preimage and Farey sequences
5. Extension in 3D
6. Duality versus polarity
7. Polygon enclosure and inclusion problems
8. Application to the convex approximation by interior.
9. Conclusion

Polygon Enclosure problem

$Enc\ (\mathcal{P}, \mathcal{Q}, \mu)$

Given a general polygon P in class $\mathcal{P}$,

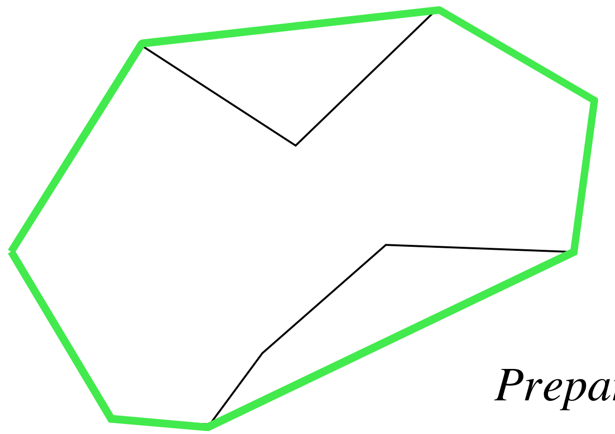
Find the μ-smallest polygon Q in class $\mathcal{Q}$

with Q containing P.

Polygon Enclosure problem

P is the class of	n-sided simple polygons
Q is the class of	convex polygons
μ is	the area measure

Enc (*P*, *Q*, μ) is the problem of convex hull with complexity in O(n log n)

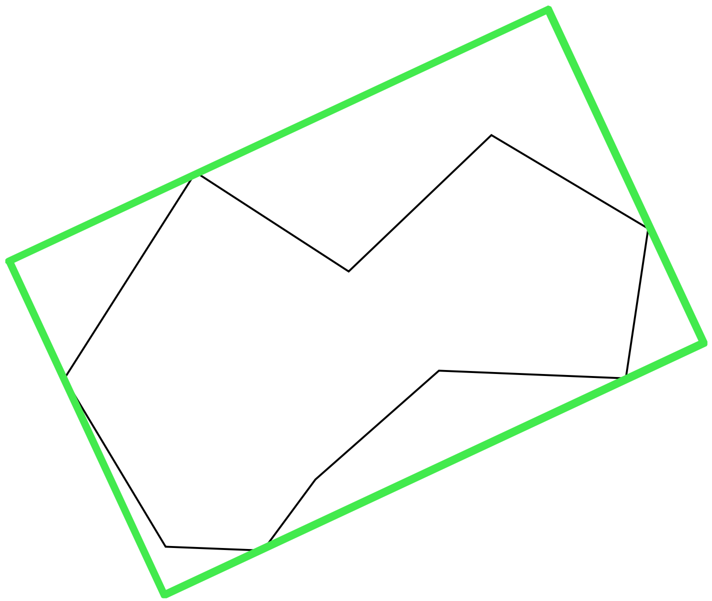


Preparata and Shamos 1985

Polygon Enclosure problem

P is the class of	n-sided simple polygons
Q is the class of	rectangles
μ is	the area measure

$Enc\ (P, Q, \mu)$ has a solution with complexity in O(n) in 2D

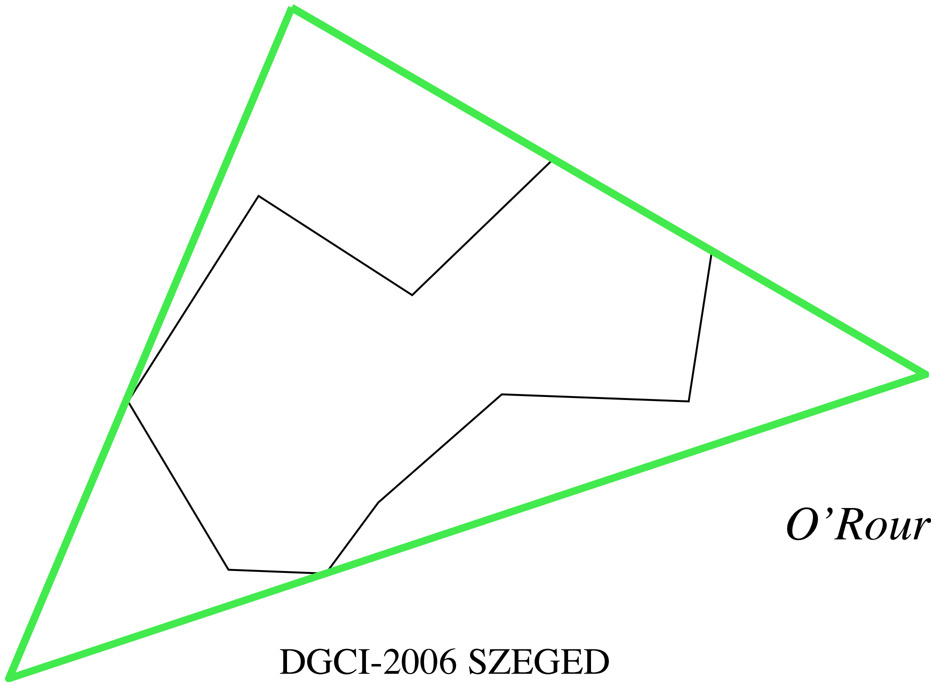


Roth and al, 1997

Polygon Enclosure problem

P is the class of	n-sided simple polygons
Q is the class of	triangles
μ is	the area measure

Enc (P, Q, μ) has a solution with complexity in $O(n \log^2 n)$



O'Rourke and al 1986

Polygon Inclusion problem

Inc (*P*, *Q*, μ)

Given a general polygon P in class *P*,

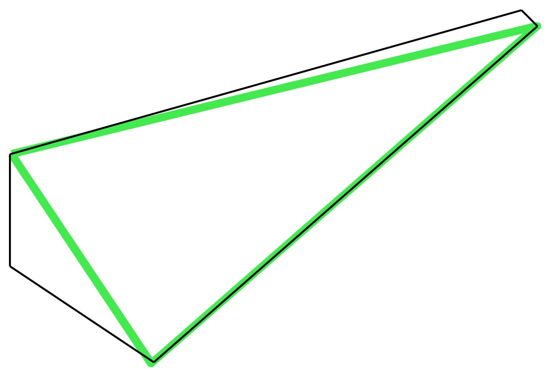
Find the μ -largest polygon Q in class *Q*

with Q contained in P.

Polygon Inclusion problem

P is the class of	n-sided convex polygons
Q is the class of	triangles
μ is	the area measure

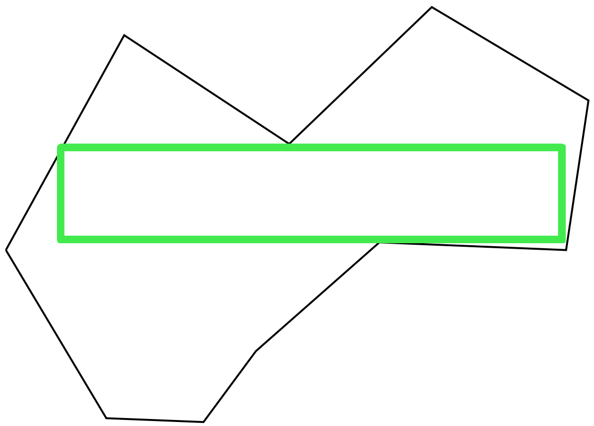
Inc (P, Q, μ) has a solution with complexity in O(n)



Polygon Inclusion problem

P is the class of	n-sided simple polygons
Q is the class of	axis parallel rectangles
μ is	the area measure

Inc (*P*, *Q*, μ) has a solution with complexity in O(n log n)

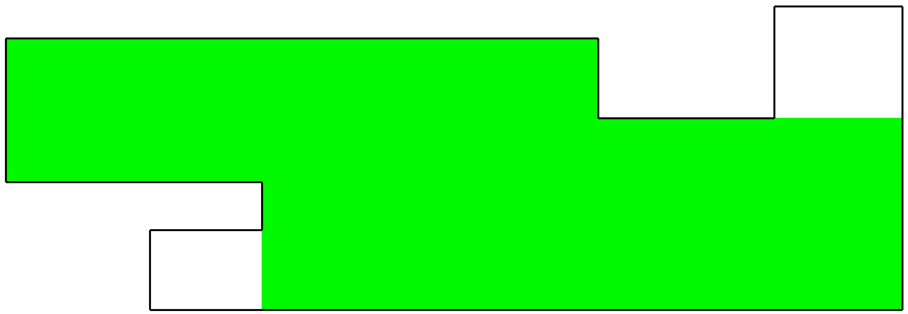


Daniels and al 1997

Polygon Inclusion problem

P is the class of	n-sided orthogonal polygons
Q is the class of	convex orthogonal polygons
μ is	the area measure

$Inc\ (P, Q, \mu)$ has a solution with complexity in $O(n^2)$



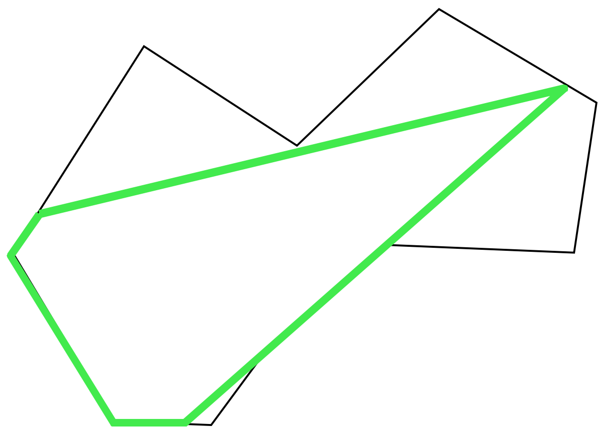
Skull problem

Wood and Yap 1988

Polygon Inclusion problem

P is the class of	n-sided simple polygons
Q is the class of	convex polygons
μ is	the area measure

$Inc\ (P, Q, \mu)$ has a solution with complexity in $O(n^7)$



Potato Peeling

Chang and Yap 1986

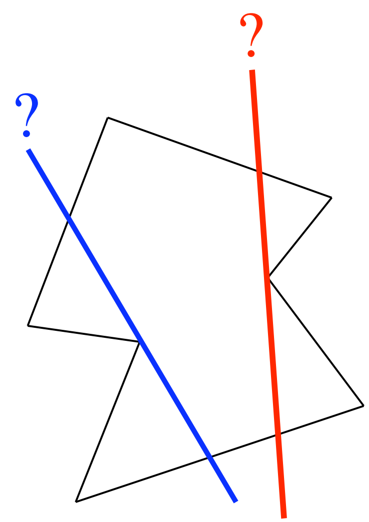
Potato peeling (Chang & Yap 1986)

$Inc\ (P, Q, \mu)$

- Property: The Maximal Area Convex Subset (MACS) included in a polygon P is a polygon (Goodman 1981).

- So the MACS will be defined as the intersections between P and a set of half-planes.

- Half-planes will be associated to reflex vertices.



Outline

1. Introduction
2. Hough transform and discrete geometry
3. Preimage of a pixel, a set of pixels, a connected straight line
4. Preimage and Farey sequences
5. Extension in 3D
6. Duality versus polarity
7. Polygon enclosure and inclusion problems
8. Application to the convex approximation by interior.
9. Conclusion

Heuristic approximation algorithm

$Inc\ (P, Q, \mu)$

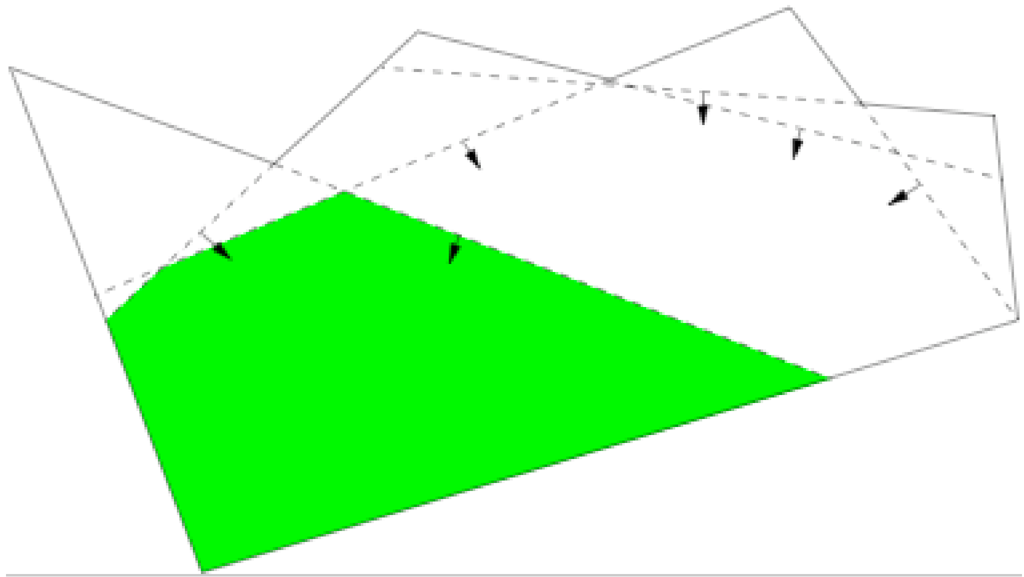
To reduce the potato peeling algorithmic complexity, we propose an approximation approach based on kernel dilation.

Chassery & Coeurjolly, ISMM, 2005

Star-shaped polygon and kernel

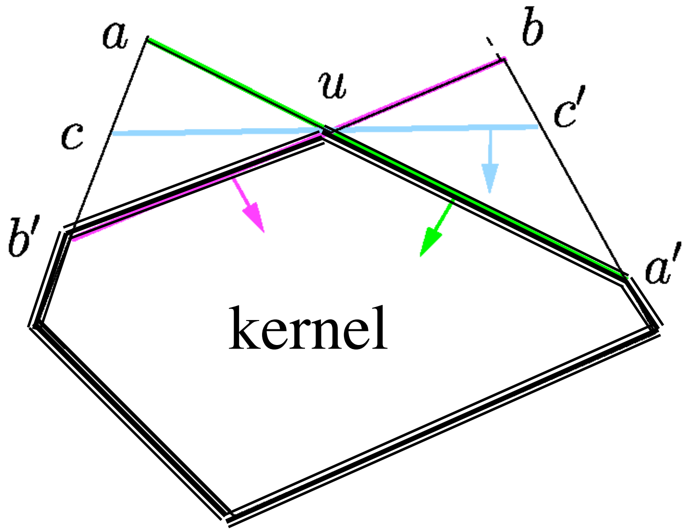
<u>Definition</u>: *P is a star-shaped polygon if there exists a point q in P such that the segment* qv_i *lies inside P for all vertices* v_i *of P. The set of such q points is the Kernel of P.*

<u>Proposition</u>: *the Kernel of P is given by intersections between P and the half-planes defined by all « extremal chords » associated to all reflex vertices.*



Kernel = dual of the convex hull of the polar polygon P

Limitation to star-shaped polygon

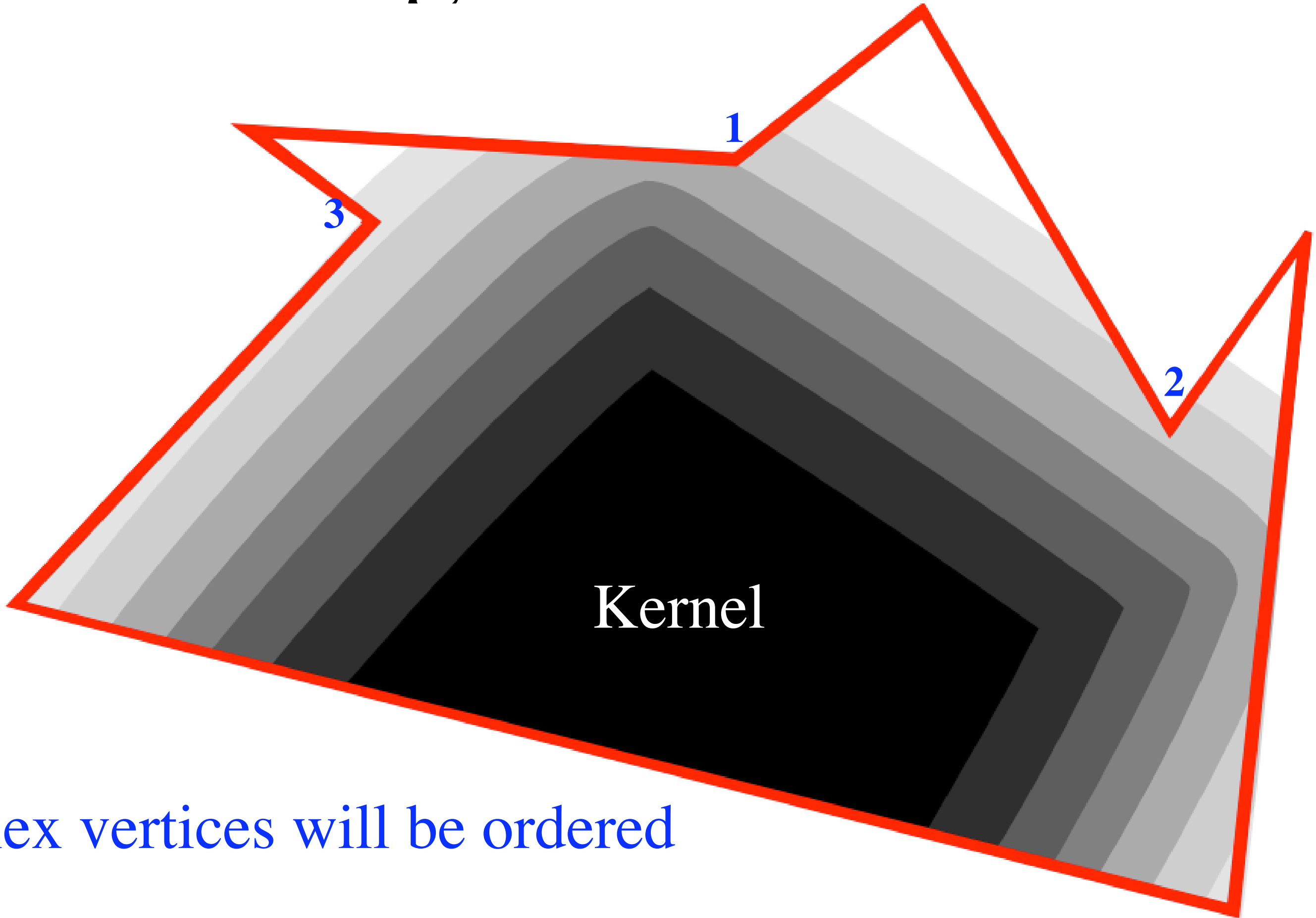


The kernel of P is a convex subset of the MACS.

How to deform the kernel to approximate the MACS ?

The deformation will be an Euclidean dilation of the kernel.

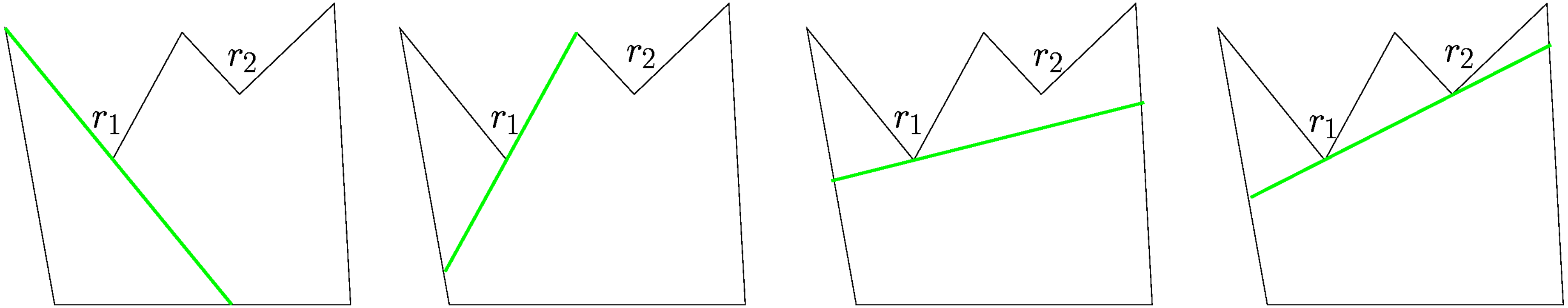
Level curves issued from kernel
Ordering the reflex vertices



The reflex vertices will be ordered

by the dilation wavefront.

Definition: A Chord is a Maximal segment fully contained in P.



(a) (a) (b) (c)

When a reflex vertex is analyzed, we have different possibilities for chords:

The chord may be an extremal chord. (a)

The chord may be a single-pivot chord (its slope is tangent to the wavefront). (b)

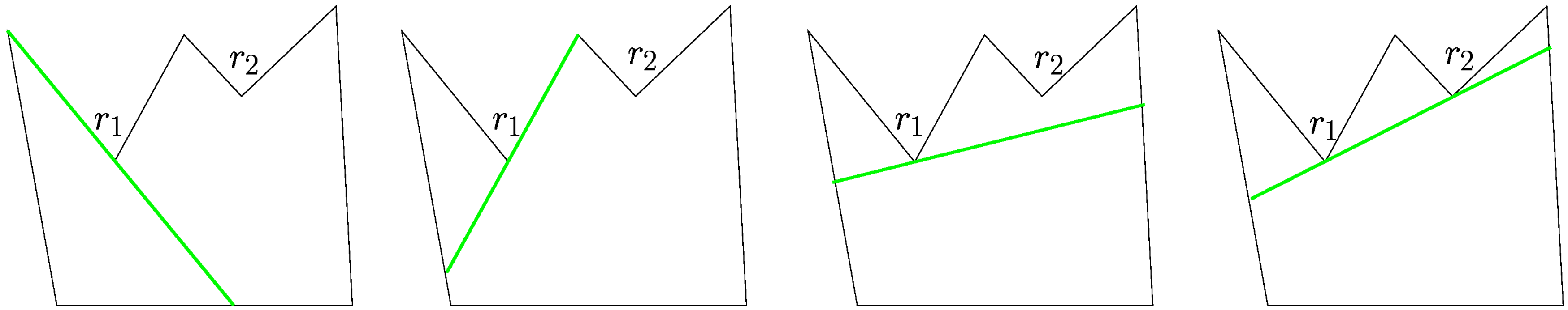
The chord may be a double-pivot chord between two successive reflex vertices. (c)

From these possibilities, we choose the chord that maximizes the area of the resulting polygon.

Approximated MACS algorithm

1. Compute the kernel of P
2. Compute the ordered list *O* of reflex vertices
3. Extract the first point r_1 in *O*
4. While *O* is not empty do
5. Extract the first point r_2 in *O*
6. Choose the best chord that maximizes the resulting polygon area with the chords (r_1, r_2)
7. Modify thr polygon P accordingly
8. Update the list *O* removing reflex points excluded by the chord
9. r_1 <--- r_2
10. End while

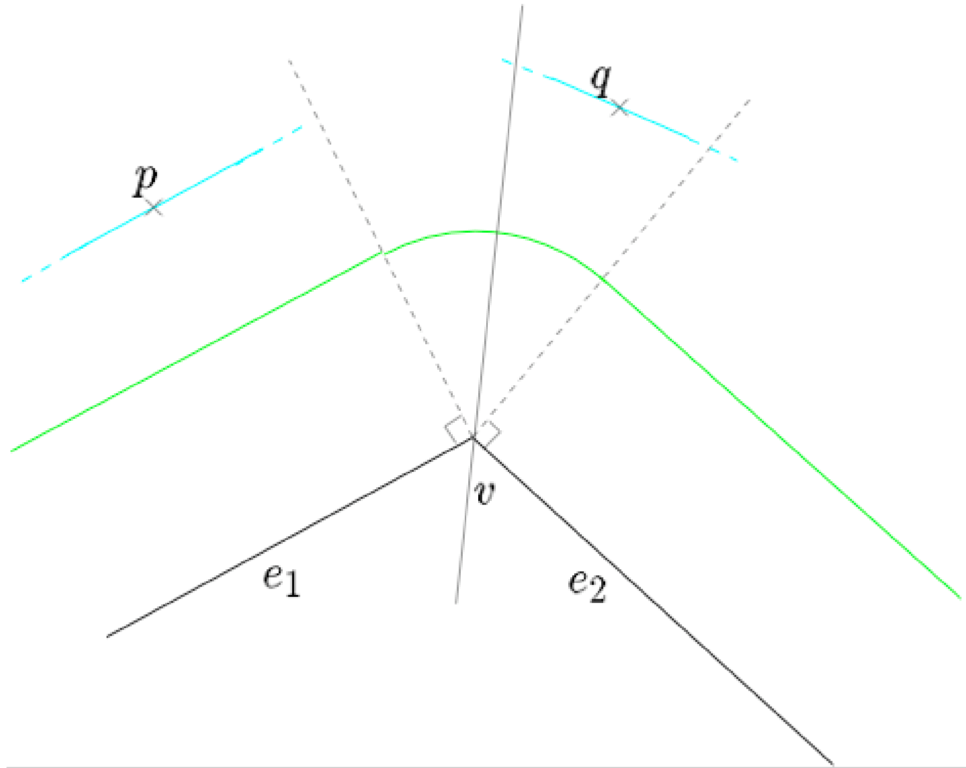
Slope computation

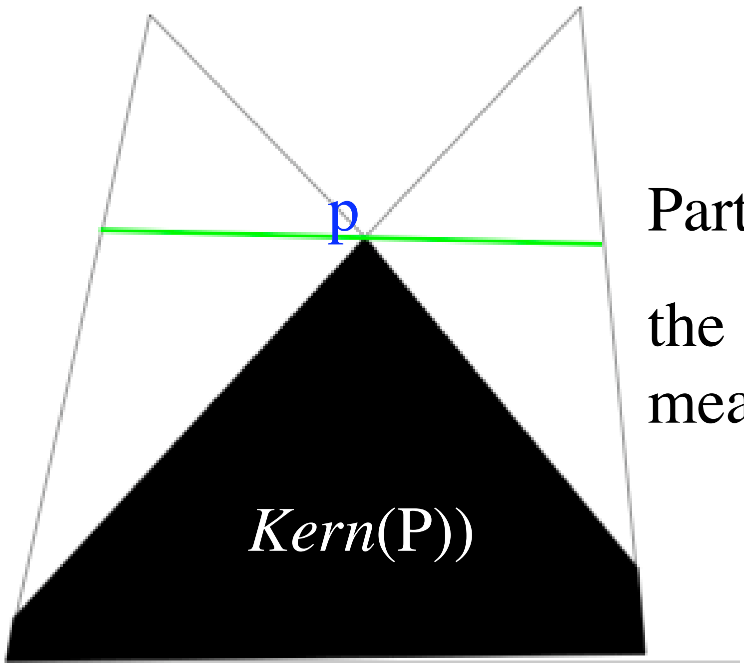


(a) (a) (b) (c)

The slope computation is direct for extremal chord (a) and double-pivot chord (c).

For a single-pivot chord :
the chord is tangent to the wavefront issued from the Kernel.

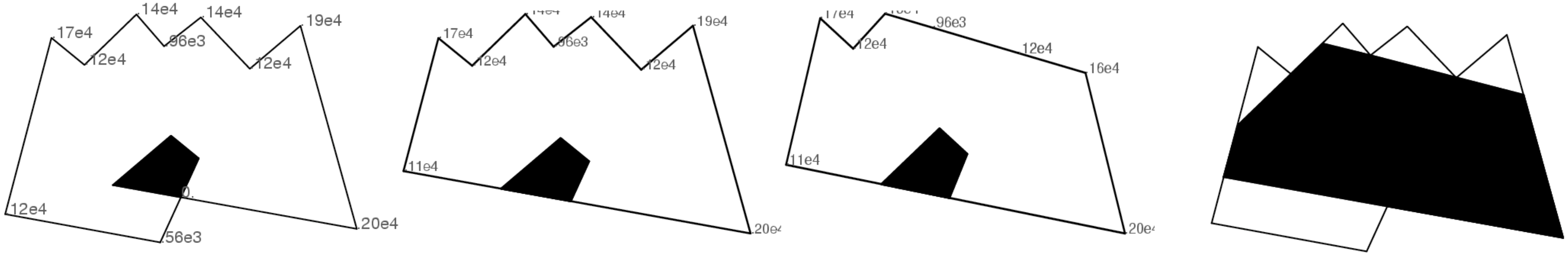




Particular case where d(p, *Kern*(P)) = 0

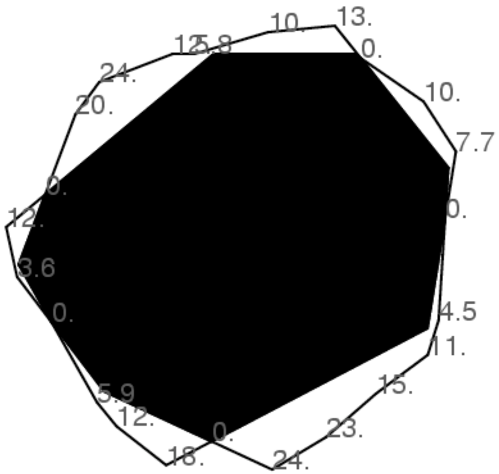
the single-pivot chord slope is obtained as the mean between the two maximal chord slopes.

Intermediate steps of the approximated MACS algorithm

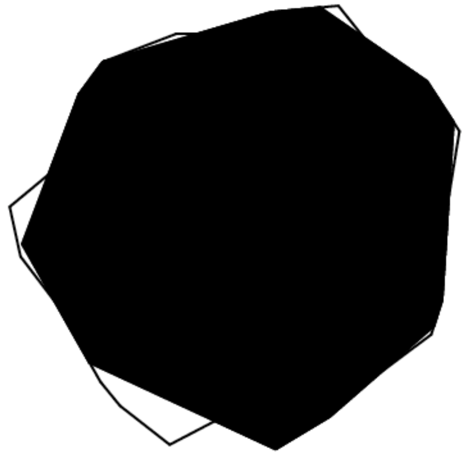


Kernel	1st cut : extremal chord	2 nd cut : double-pivot chord	3 rd cut : extremal chord

Results

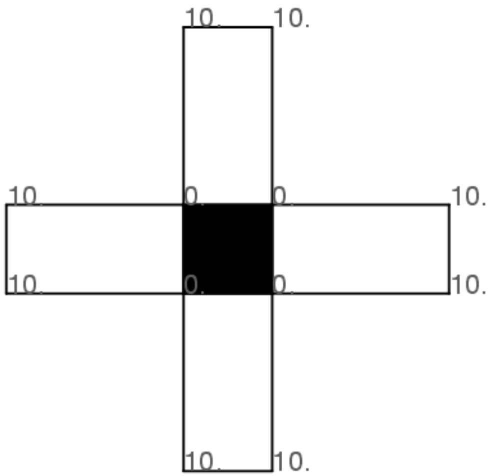


Initial polygon and its kernel

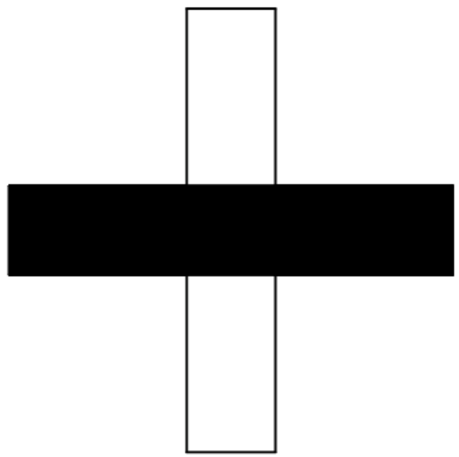


Approximated MACS

Results

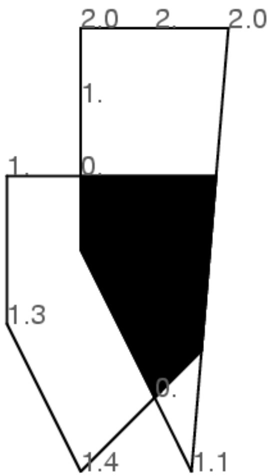


Initial polygon and its kernel

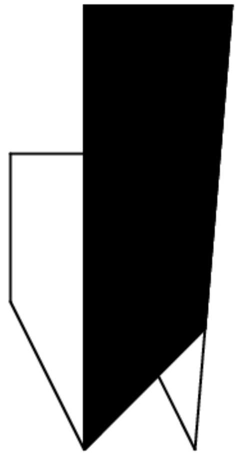


Approximated MACS

Results

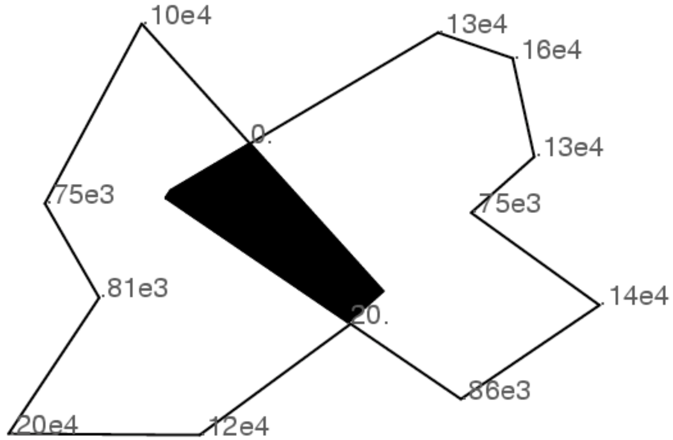


Initial polygon and its kernel

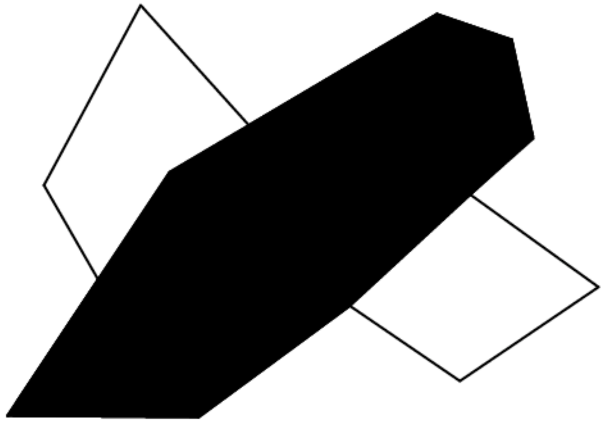


Approximated MACS

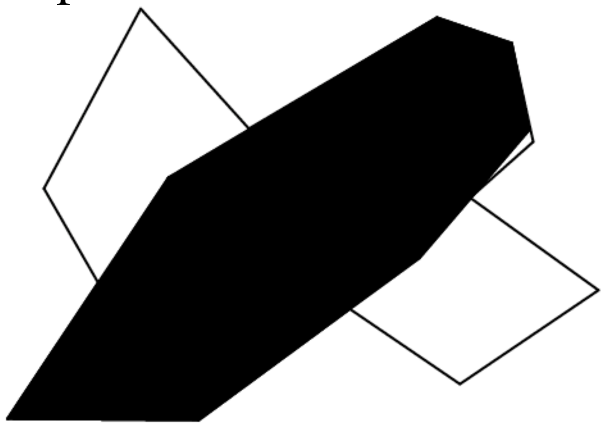
Comparison between Optimal MACS and Approximated MACS



Initial polygon and its kernel

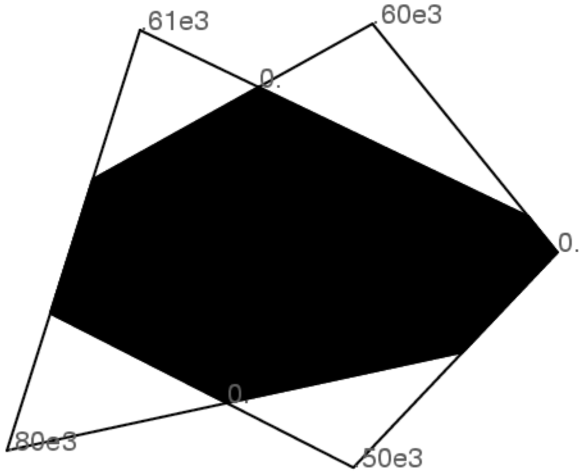


Optimal MACS

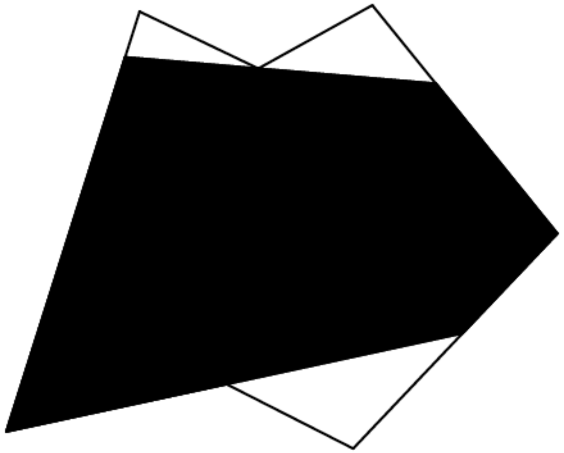


Approximated MACS

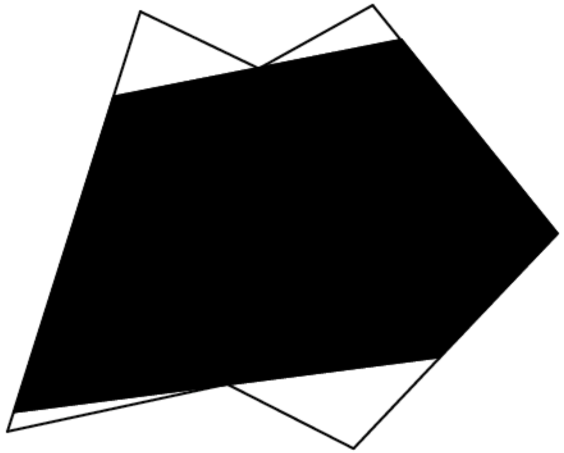
Comparison between Optimal MACS and Approximated MACS



Initial polygon and its kernel

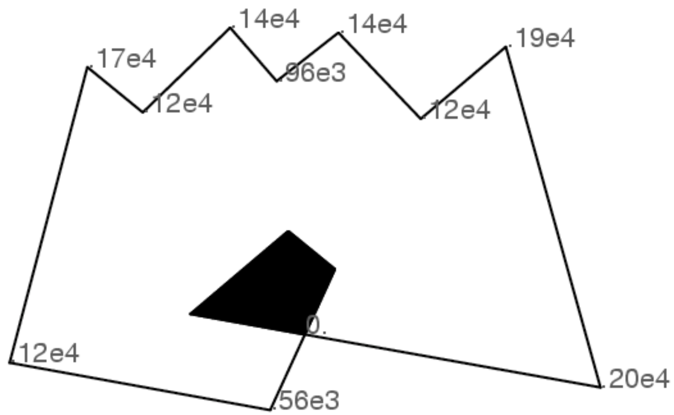


Optimal MACS

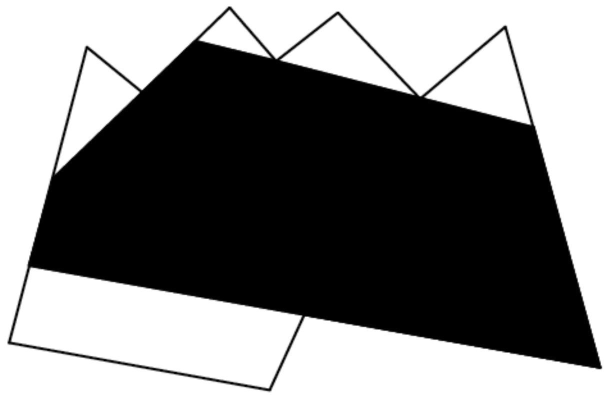


Approximated MACS

Comparison between Optimal MACS and Approximated MACS



Initial polygon and its kernel



Optimal or Approximated MACS

Computational cost

n : number of vertices of P

k : number of reflex vertices

Complexity in O (n + k log k)

1. Compute the kernel of P	• O (n)
2. Compute the ordered list *O* of reflex vertices	• O (k)
3. Extract the first point r_1 in *O*	• O (k log k)
4. While *O* is not empty do	
5. Extract the first point r_2 in *O*	
6. Choose the best chord that maximizes the resulting polygon area with the chords (r_1, r_2)	• O (k)
7. Modify the polygon P accordingly	• O (k)
8. Update the list *O* removing reflex points excluded by the chord	
9. r_1 <--- r_2	
10. End while	

Outline

1. Introduction
2. Hough transform and discrete geometry
3. Preimage of a pixel, a set of pixels, a connected straight line
4. Preimage and Farey sequences
5. Extension in 3D
6. Duality versus polarity
7. Polygon enclosure and inclusion problems
8. Application to the convex approximation by interior.
9. Conclusion

Conclusion

Two approaches for duality:

- parametric transform
- geometric transform

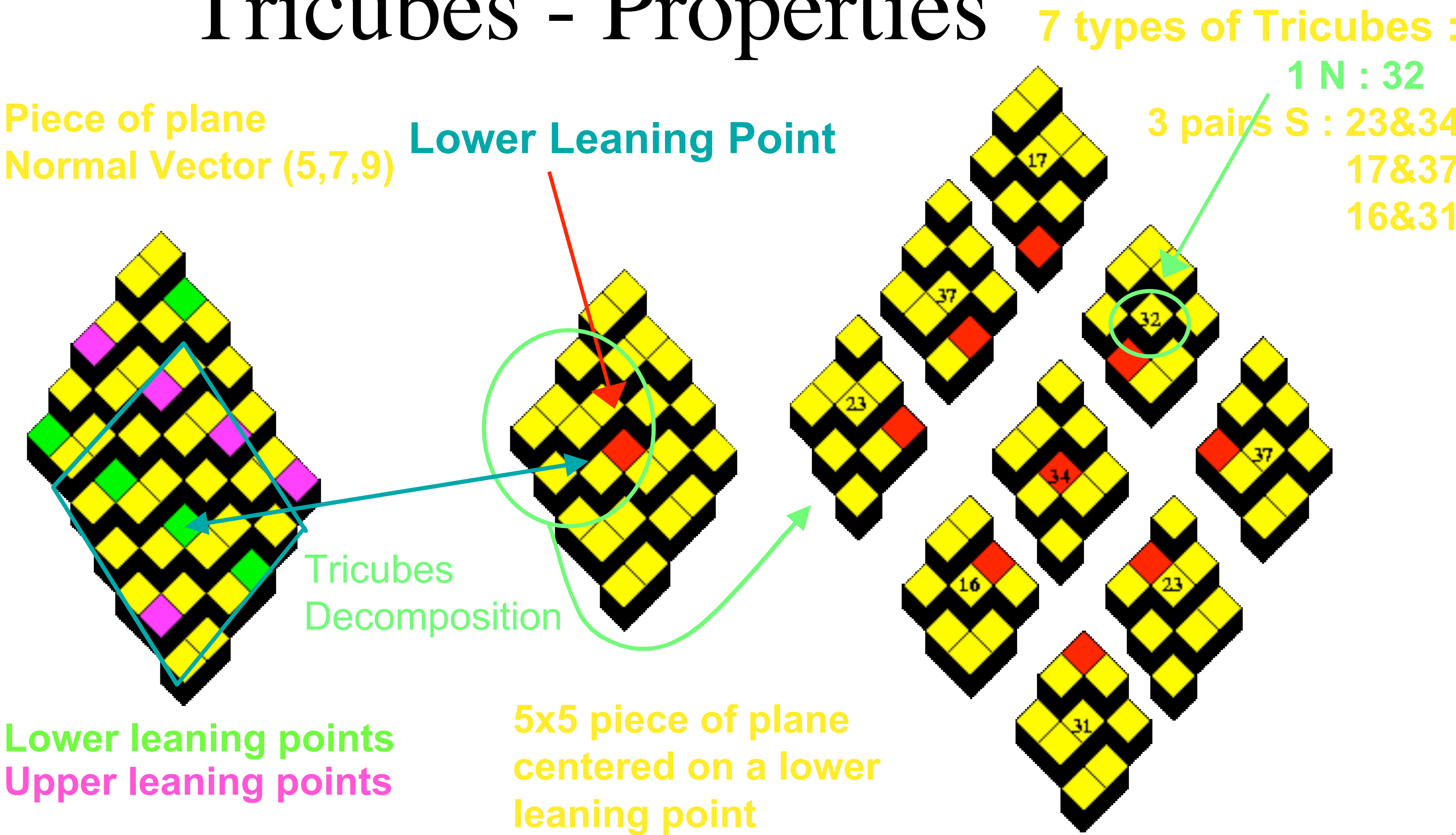
Duality offers new representation space for

- characterization,
- recognition,
- optimization.

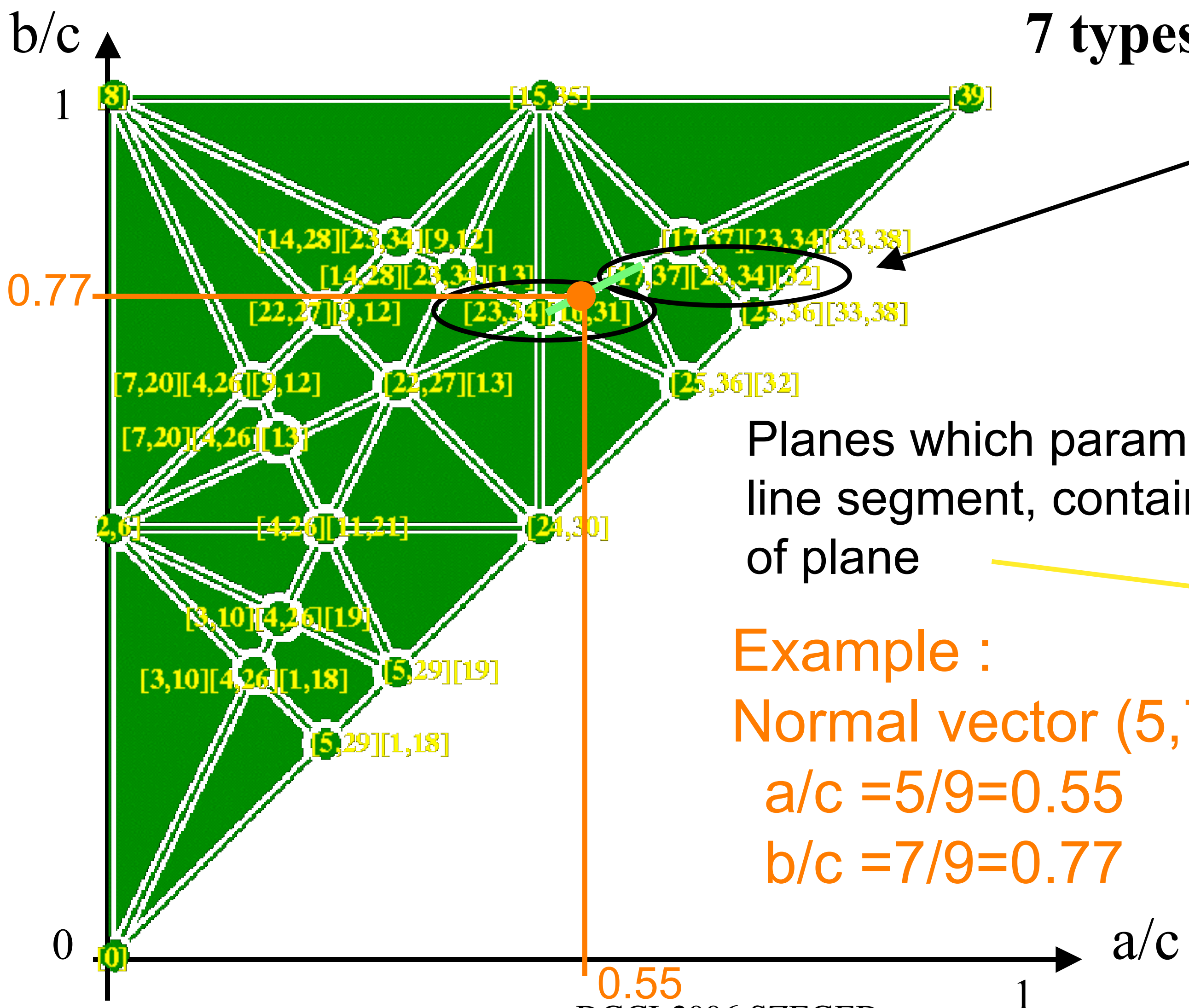
Duality versus polarity has been illustrated.

Such work may be extended.

Tricubes - Properties



Farey diagram associated to tricubes



7 types of Tricubes :

1 N : 32

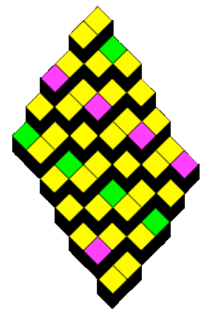
3 pairs S :

23&34

17&37

16&31

Planes which parameters are on this line segment, contain the initial piece of plane



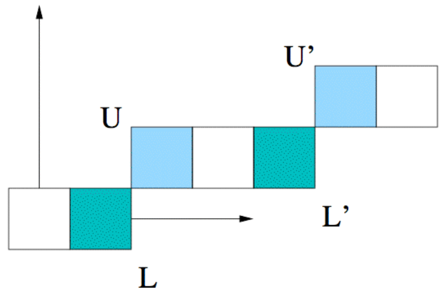
Example :

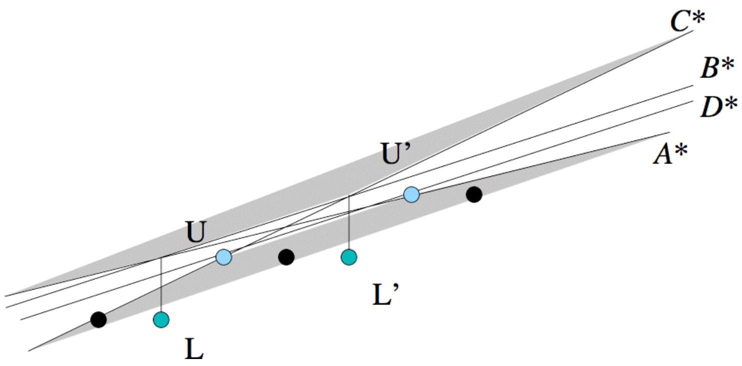
Normal vector (5,7,9)

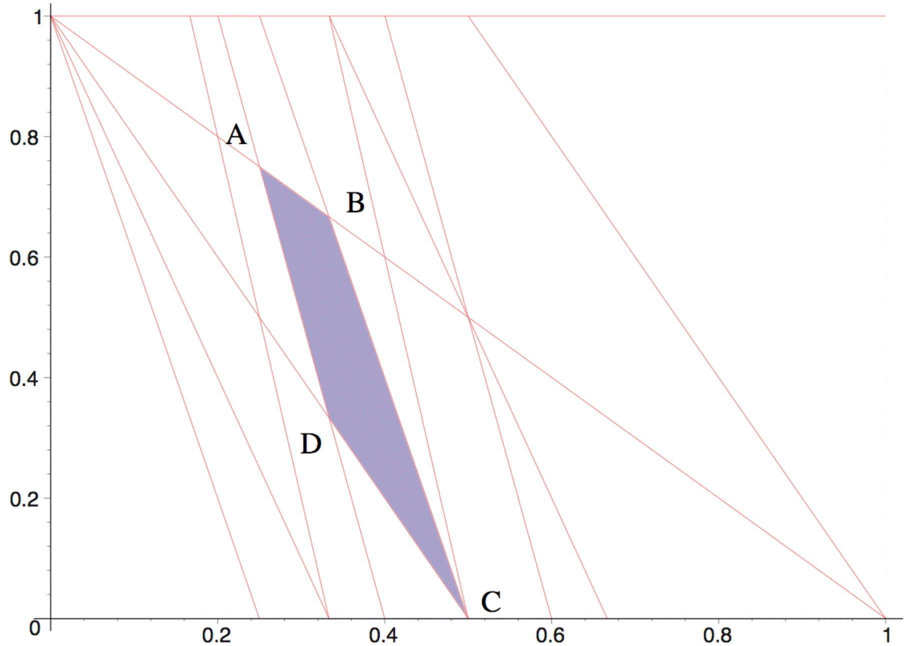
a/c =5/9=0.55

b/c =7/9=0.77

Relation between digital straight line segment and duality



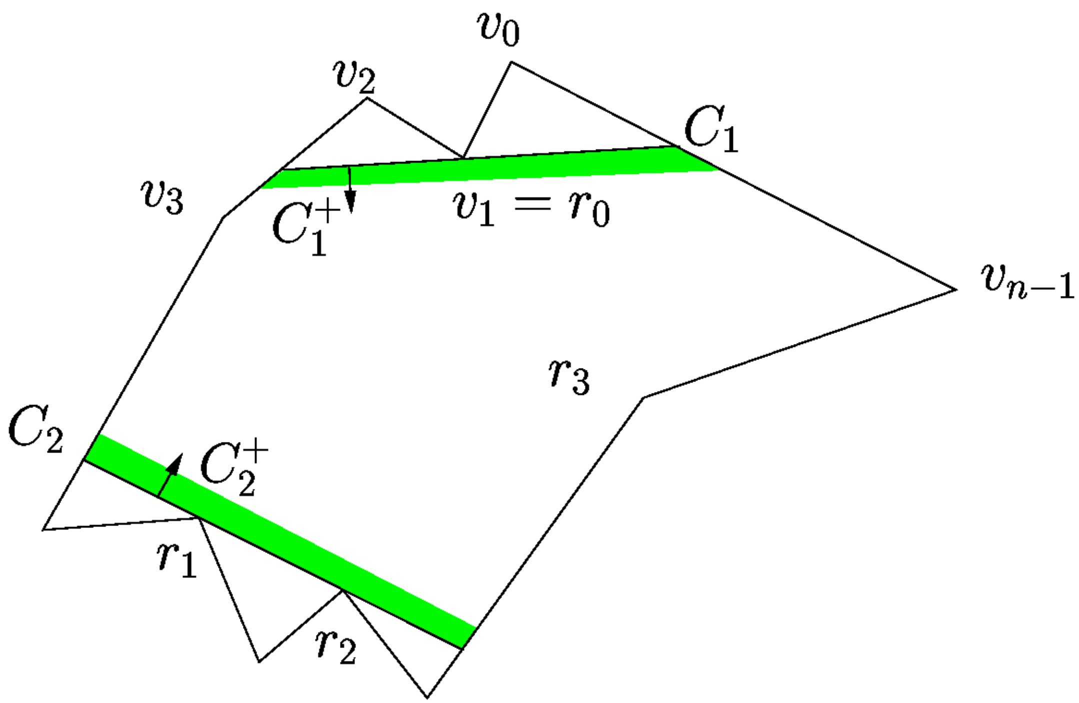




Optimal solution is based on finding constraints on different reflex vertices:

- in case of one isolated reflex vertex --> an infinity of solutions (slope of the constraint)

- in case of consecutive reflex vertices --> one solution



Optimal solution is obtained by intersection of all the constraints.